TANABE

A Comparison of the Kelvin Double Bridge and the Potentiometer Methods of Measuring Low Resistances

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A COMPARISON OF THE KELVIN DOUBLE BRIDGE AND THE POTENTIOMETER METHODS OF MEASURING LOW RESISTANCES

BY

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B.S. Knox College
1911

THESIS

Submitted in Partial Fulfillment
of the Requirements for the

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

STETFAN TANABE

ENTITLED A COMPARISON OF THE KELVIN DOUBLE BRIDGE AND THE POTENTIOMETER METHODS OF MEASURING LOW RESISTANCES

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INTRODUCTION

In recent years, by utilizing water power and much improved, efficient steam engines, and also very efficient continuous and alternating current, electric generators, the cost of producing electricity has been greatly reduced. In almost every avenue of life today, in North America, electricity is used in one form or in another, such as light, heat and power: it is a necessity.

With such advancement in generating and utilization of electricity, there are corresponding large demands for meters and instruments to measure electrical quantities, directly or indirectly. These instruments, measuring electricity, are used either for the purpose that power plants may charge their customers for the amount of electricity consumed or in order to test electrical machinery after completion or after installation.

Such meters and instruments are tested and calibrated, as to their indications, from time to time, at various laboratories. Some of these laboratories are maintained by state or federal governments; some are established by power supply stations in large cities, while still others are under the management of manufacturers of electrical machinery and equipment.

At present there is a general tendency in electrical engineering to build and run generators of large capacity; also there are increasing demands for powerful motors. It becomes necessary to find means to measure high voltage and also to measure large current.

In the measurement of current it is costly to manufacture ammeters, whose coils are capable of carrying enormous current. Such ammeters, even when manufactured at low cost are necessarily bulky and are cumbersome to handle. Accordingly current transformers are used

in cases of large alternating currents with ammeters and it is customary also to use shunts for continuous currents of large magnitude in connection with ammeters or milli-voltmeters for measurements.

I Demand for Measurements of Low Resistances Using Large Current

It is well known to those who are familiar with electrical engineering that when an ammeter is provided with a shunt, it is a millivoltmeter which indicates the small voltage across the shunt through which a current is flowing. The calibrations of such ammeters or millivoltmeters involves the correctness in the resistance of its shunts besides proper indication upon the scale of the instrument.

The ratio between the currents in the primary and secondary windings of a current transformer is tested in the laboratories. In present practice, checking the ratio of the two windings is made not by comparing the currents in the two windings directly, but indirectly by the inverse relation of a known resistance inserted in the primary circuit to another known resistance inserted in the secondary circuit, as given by a formula *

$$\frac{I'}{I''} = \frac{R''}{R'} \sqrt{1+\frac{2\pi f M}{R''}}$$

Where I' is the current in the primary, I" the current in the secondary, R' the known resistance in the primary, and R" the known variable resistance in the secondary, while

$$\sqrt{1 + \frac{2\pi f M}{R''}}$$

is a correction term.

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* E. T. Robinson, "Electrical Measurements in Circuits Requiring Current and Voltage Transformer", Proc. A.I.E.E., Vol. 28, Part 2, page 1005, 1909.

C. H. Sharp and W. S. Crawford, "Some Recent Developments in Exact Alternating Current Measurement," Proc. A.I.E.E., Vol. 29, Part 2, p. 1517, 1910.

These known resistances R" and R' are called current carrying shunts.

Therefore measurements of direct or alternating currents of large magnitude are ultimately based upon the value of the shunts. The correctness of indication on the ammeter scale depends upon the correctness of resistance of the shunts in large continuous or alternating current measurement work.

A certain accuracy is demanded in any experiments or in tests of measurements in general. In our present state of knowledge, it is not possible to measure or give absolute value. Whatever we read off on scales or measure by instruments a certain amount of error is involved in our reading. Some of the errors are inherent in the instruments used, while some are due to faulty observations. In determining the ratio of current transformers and in the calibrating instruments, an accuracy, which allows total errors of only 1/1000 to 2/1000, is demanded. The erors of 1/1000 to 2/1000 are divided into graduation of scales, determining resistance of the shunts, observation and calculations. In order to fulfil this requirement, the value of the shunts must be determined by a method involving 1/10000 errors at the maximum, and difference of 1/10000 of the value can be detected by it. Hereafter in this paper instead of saying so many parts of the value sought when accuracy is discussed, percentage expression is used, such as 1/10000 of the value is expressed 0.01%.

The shunts which are used in continuous current circuits, are generally made of aged manganin sheets connected in parallel between copper blocks, while the shunts which are employed in alternating current circuits , are made of the same material but constructed in such a way that the self-inductance of a shunt is a minimum.

Some of these shunts made for high current capacities are immersed in an oil bath, and the oil in turn is cooled by running water, while in other cases, such shunts are cooled by running water alone. Hence, the resistances of the shunts are made as constant as possible in virtue of small temperature coefficient and good constancy of manganin, and of the holding the temperature of the shunts in operation from rising by cooling devices. The current carrying capacities of the shunts range from one ampere to ten thousand amperes, while their resistance varies from 1 ohm down to 0.00005 ohm.

In the laboratory with which I am connected a number of these shunts are constructed and are used in the calibration of ammeters and in testing the ratios of current transformers. In order to keep a close watch on the permanancy of the resistances of the shunts, these shunts are measured at regular intervals besides special measurements are made on then when there arises any suspicion of change in resistance in any one of them during the calibration of meters or in current transformer testing.

Besides the above mentioned shunts, there are a number of other shunts or resistance "standards" of low value used in various other tests upon electrical machines. These also must be measured from time to time to assure their dependability.

Copper, aluminum, other metals and alloys of various combinations are brought to the laboratory to be tested for conductivity or for resistivity. The samples of such metals and alloys often are in form of rectangular bars or cylindrical rods. The resistance of some of these samples may be as low as 0.00001 ohm.

Therefore, the resistances of all these shunts and standards, and metals and alloys are one ohm or less. The Wheatstone bridge

method cannot be used upon them, to measure their value to any satisfaction. Hence other methods of resistance measurements either involving a deflection or a null reading must be sought for. As the value to be measured is so low a large current must be used in order to increase the sensibility of the measurements. It is also desirable, if possible, to use the full current, for which a particular shunt is designed in order to reproduce the same condition as far as possible under which the shunt is used, so that heat effect, if there be any, can be observed during resistance tests.

In the laboratory, in determining the resistance of various shunts, the continuous current only is used. The source of the current is a battery of storage cells of large capacity or a generator which was especially constructed to furnish large currents.

In a commercial or industrial laboratory such as this place is, all the measurements must be made with fair accuracy and with minimum expenditure of time. The accuracy demanded in measuring the resistances of the shunts of various types and those of the diverse metals and alloys is from 0.02% to 0.05%.

It is the object of this paper to show the method or methods suitable in such resistance measurement work.

In the laboratory, the basis of all the resistance comparisons are standards of resistance ranging from 1 ohm down to 0.0001 ohm whose value was previously determined with high precision. It is beyond the scope of this paper to discuss or describe how the values of these standards of resistance are determined.

II The Method Employed in General for Measurements of Low Resistances

Since the resistance of the shunts in question and that of metals and alloys are low, the requirement for measurement is an accuracy within 0.02% to 0.05% and with rapidity, and so only methods which answer the foregoing requirements are considered. To insure rapidity, direct readings, ease in manipulation, and simple computation are very desirable factors in the selection of the method.

The methods used in measuring low resistances in various laboratories may be divided into two general groups, namely,

1 Deflection Methods

2 Null Methods

I Deflection Methods.

The deflection methods are those in which the deflection of the galvanometer is the main factor. The methods which involve ammeter readings will be discarded because no ammeter, not even a standard laboratory ammeter, is expected to indicate a current accurate to within 0.1% and any error in the observation makes the accuracy still less. The deflection methods depending upon formulae, derived by approximation, are also out of consideration. These eliminations leave the deflection methods that depend on resistance in the circuit besides the deflection of a galvanometer.

The resistance of unknown value may be found when it is compared against a resistance standard and the difference between the two is evaluated from a galvanometer deflection. The connections of this method are shown in Fig. 1 and the formula is

$$R = S\frac{d_2}{d_1}$$

where S is the resistance of the standard, d_1 is the deflection of the

galvanometer due to potential drop over S, d_2 is the deflection due to potential drop over the unknown. Then R is the resistance sought. The best conditions for this method are when d_1 and d_2 are nearly alike, and are nearly 250 scale divisions. Even then the difference of 1 part in 1000 is the best that may be expected. The unequal graduation of the scale and the creeping of the galvanometer also augments errors. Such methods cannot therefore be used in high precision measurements. The writer made a modification of this method.

In order to minimize contact resistance and lead wire resistances, resistances in the galvanometer are increased. To make the current through the galvanometer nearly the same when either the unknown or the standard is connected to the galvanometer, a resistance is inserted in one of the lead wires from the standard which is always a higher resistance than the unknown. The connection is shown in Fig. 2. The working formula is

$$R = S\frac{d_2}{d_1} \cdot \frac{G+g}{G+g+S}$$

Where G+g is several hundreds or even several thousands of ohms so that the divided current does not appreciably affect the main current. By varying S, $\frac{d_2}{d_1}$ may be made unity, and then the formula will be simplified to

$$R = S\frac{G+g}{G+g+S}$$

In this way we can detect a difference of five parts in 10000 easily. However, the errors arising from the plug resistance boxes used for S and g, make an accuracy within 0.5% impossible and the method if not fitted for exact measuring.

II Null Method

It is generally understood that a null method means that no current flows through the galvanometer when the two terminals of the

instrument are at the same potential. It is, however, a fact that th there may be some current flowing through the galvanometer. The magnitude of the galvanometer current may be very small. The amount of such current depends chiefly upon the sensitiveness of the galvanometer in use and upon the eye-sight of the observer. Supposing the sensitiveness of the galvanometer is 1×10^{-8} amperes per millimeter deflection and the observer can detect 2/10 millimeter deflection, then the deflection that may just escape observation is caused by a current less than 2×10^{-9} ampere.

There are several null methods that are used in precision resistance measurements of low value. They are

a) Carey-Foster bridge
b) Differential galvanometer method
c) Matthiessen and Hockins bridge
d) Hausman bridge
e) Two-step method by A. Campbell
f) Kelvin double bridge
g) Potentiometer method.

(a) The Carey-Foster bridge methos is well known and it is suitable for precise resistance measurements. See Fig. 3. The formula is;-

$$S_1 = S_2 + (a_2 - a_1).$$

The method is admirably suited for resistances of half an ohm and upwards. The method, however, demands a certain construction of the resistances to be tested. The size of the slide wire limits the amount of current to be used. Hence it cannot be used to advantage in low resistance work.

(b) The differential galvanometer method can be subdivided in two parts:

The first of these is the deflection method

$$R = S\frac{d_2}{d_1+d_2}$$ See Figs. 4 and 5.

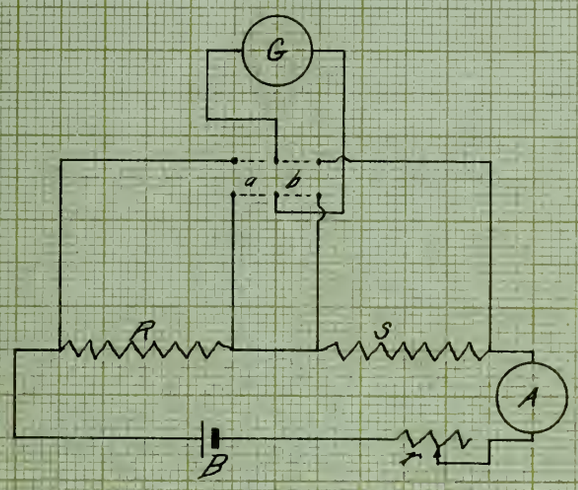
that is to say, the value of R, the unknown, may be evaluated from deflections by varying S, the standard of resistance.

There are two kinds of galvanometers of this type, the same as in other galvanometers, namely, the moving coil type and the moving needle type. In each of the types there always exists lack of symmetry in the galvanometer coils, so that the deflection given is not proportional to the difference in the currents of the two galvanometer circuits, although magnetic balance may be secured by adjusting one coil or the other, when the position of the coil is in the center of the magnetic field. Such defects may be overcome by calibration of the galvanometer deflection from time to time. When the difference between the value of S and that of R is large, this method gives rather a large error.

At this point it should be pointed out that the moving needle type of galvanometer is not suitable to use in the place where variable stray magnetic fields exist, even though the galvanometer is well shielded. It gives trouble in such places. The moving coil type of galvanometer is provided with a strong permanent magnetic field, and hence outside stray fields affect it but feebly.

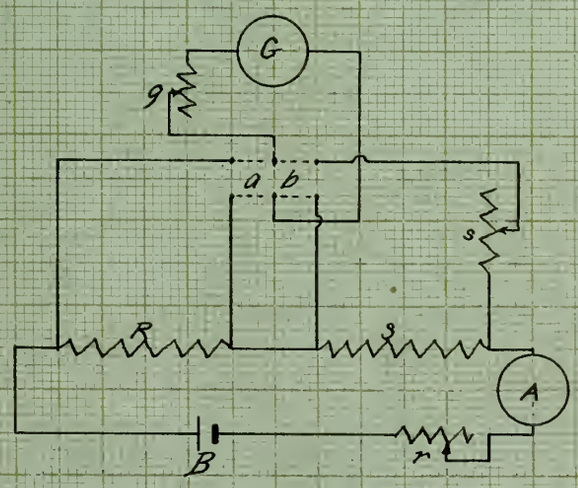
The Second Null Method. The arrangement of this isshown in Fig. 4 and 5. The magnetic balance can be obtained by either introducing "g" the resistance in one of the galvanometer coil circuits, G' and G", or shunting one of them. By balancing, interchanging R and S, by adjusting or varying S till there is no deflection. The value of R is determined by $R = \sqrt{S_1 S_2}$, where S_1 and S_2 are shunted values of S. When the resistances to be compared are low, we use shunt S instead of varying itself, in order to secure zero deflection and use the connection shown in Fig. 4, instead of Fig. 5. The "set-up" in Fig. 4 is called the Heaviside modification and it makes the

Fig 1
Deflection Method
$R = S \frac{d_2}{d_1}$



G – galvanometer
A – indicating ammeter
r – rheostat
S – standard resistance
R – resistance being measured
a b – double pole, double throw switch

Fig. 2
L.W. Wild Method
$R = S \frac{d_2}{d_1} \cdot \frac{G+g}{G+s+g}$



s – plug resistance
g – galvanometer series resistance, to make total of G circuit a multiple of 10.

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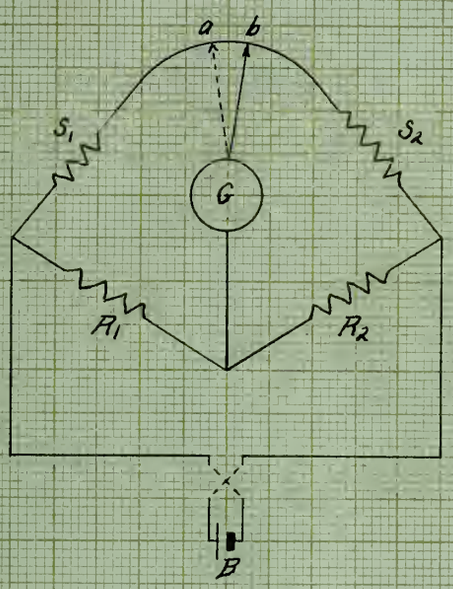
Fig 3



Carey Foster Bridge

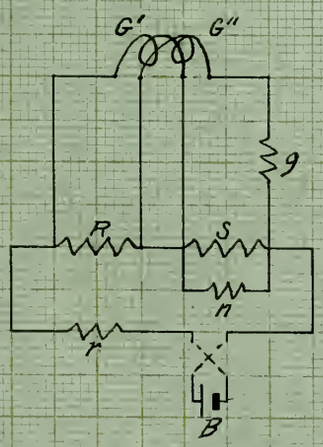
$S_1 = S_2 + \rho(a_2 - a_1)$

S_1 & S_2 are resistances to be compared.

ρ = resistance of slide wire per mm.

Fig. 4

Differential Galvanometer Method.



By deflection method.

$$R = S + \frac{d_1}{d_1 + d_2}$$

By null method.
Interchanging R & S.

$$R = \sqrt{S_1 S_2}$$

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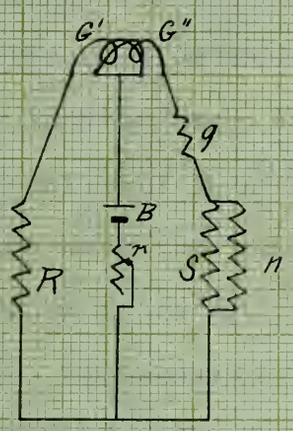
Differential Galvanometer Method

Fig. 5



By deflection method

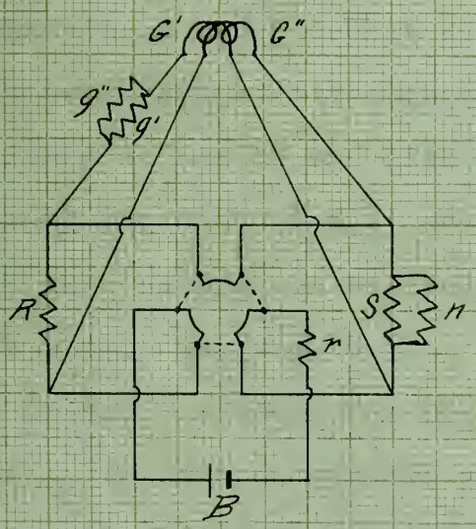
$$R = S + \frac{d_1}{d_1 + d_2}$$

By null method
Interchanging R + S
$R = \sqrt{S_1 S_2}$

S_1 + S_2 are shunted values of S

Kohlrausch Differential Galvanometer Method.

Fig. 6



g'g" = variable resistance to obtain magnetic balance of galvanometer coils

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galvanometer more sensitive when it has a resistance of several hundred ohms.

When R is nearly the same as S and the galvanometer resistance is suitable for the particular circumstance and the direct reading in shunting S is available instead of making calculations, the differential galvanometer null method can be used to great advantage. It can be speedily manipulated and the limit of current used is determined only by the capacity of R or S, or both.

In the case when the relation of R to S is very far from unity, say R to S is 1 to 2, or 1 to 3, then R and S cannot be exchanged in their positions, conveniently, for in that case galvanometer resistance must be changed. In such cases a substitution method must be resorted to. The formula will be

$$R_2 = \sqrt{\frac{S_2\ S_3}{S_1\ S_2}} \cdot R_1$$

Where R_2 is the unknown, R_1 is the standard while S_1, S_2, S_3, and S_4 are the shunted value of another standard S.

Exchanging R and S causes a difference of resistance in the two galvanometer circuits, although slight, and this introduces certain errors. When G' and G" are high resistances, these errors will be small but at the same time it makes the system less sensitive. The errors of this nature can be avoided if Kohlrausch over-wrapping method if used. See Fig. 6.[1]

The writer, however, has not performed enough experiments, up to the present, to express his own views upon the method. The method is not used in this country and although it is the method for low

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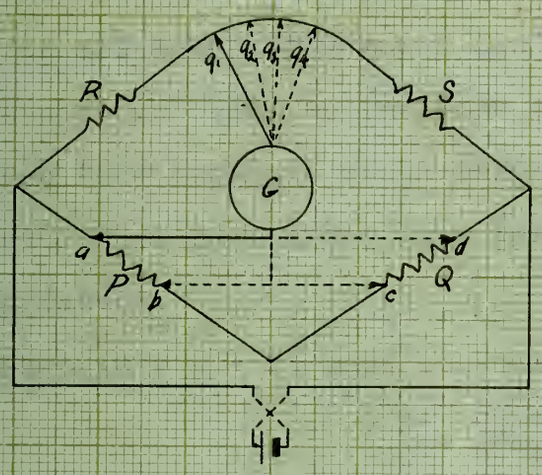
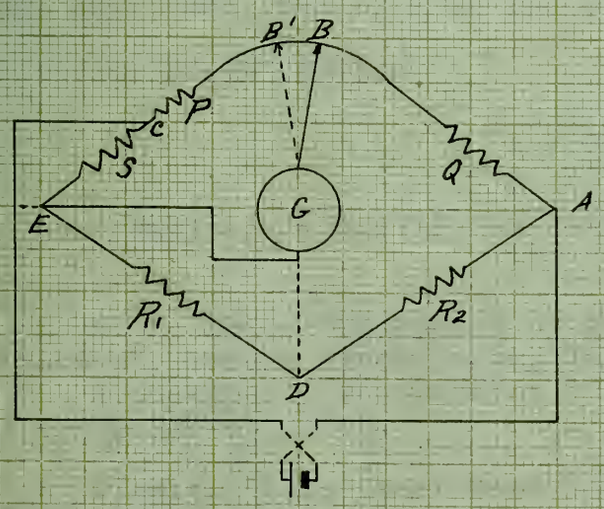
1. W.J. Jaeger "Differential Galvanometer", Zeitschrift für Instrumentenkunde, Vol. 24, p. 288, 1904.

Fig 7
Mathiessen & Hockins Bridge



$$\frac{P}{Q} = \frac{q_1 q_2}{q_3 q_4}$$

Fig. 8
Housman Bridge



$$P + Q = r$$

$$R_2 = \frac{Q\,Q'}{P(S+r)}\,S$$

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resistance measurements in Germany. The laboratories in this country are not inclined to adopt it readily. According to Mr. A. Campbell and Mr. E. E. Smith the sensitiveness of the differential galvanometer is 0.098 i $\frac{dR}{R}$. The value of current i must not be large enough to alter the value of G' and G".[2]

(c) Mathiessen and Hockin's Bridge. Fig. 7. In this method the value of all leads to Q and P can be eliminated and a large current can be used in order to increase sensitiveness. The method, however, requires four separate steps before the resistance of P can be calculated. This fact alone is a great drawback when rapidity is a required factor.

When $(R + s + S)$ kept constant

$$\frac{P}{Q} = \frac{q_1\ q_2}{q_3\ q_4}$$ [3]

(d) Hausman Bridge. Fig. 8. In using Hausman's bridge method, two balances must be made. By the first balancing R_2 is compared against $R_1 + S$; in the second balance R_2 against R. Since the value of S is one thousand or ten thousand times R_2, and ten or hundred times R. The current used is therefore limited by the capacity of S in one case, and that of R in the other. This condition reduces the sensitivity of the method.

$$R_2 = \frac{QQ'}{P(S+R)}$$ [3]

Lord Rayleigh modified the former method so that it can be used

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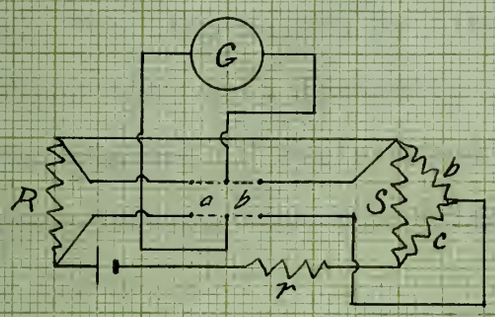
2 E. E. Smith "On Method of High Precision for Comparison of Resistance", London Electrician, Vol. 57, pp. 976-78, 1009-1011.
A. Campbell "Measurement of Small Resistance", Phil. Mag., Vol. 6, pp. 33-41, 1903.

3 Fleming "Laboratory Hand Book on Electrical and Magnetic Measurements", Vol. 1.

Fig 9

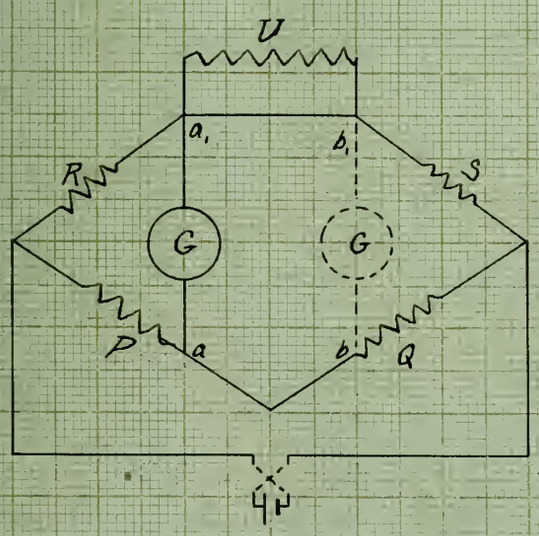
Lord Reyleigh's Modification of Housman Bridge.



$$R = \frac{Sb}{S+b+c+\frac{bc}{G}}$$

Fig 10.

Two Step Method by A. Campbell.



as a deflection method with an ordinary galvanometer or a null method with a differential galvanometer. See Fig. 9. By so doing one value of current is used flowing through S and R.[4]

$$R = \frac{sb}{s + b + c + \frac{bc}{G}}$$

(e) Two Step Method by Albert Campbell. Although Mr. A. Campbell claimed that this method is very sensitive, it is extremely tedious when numbers of resistances must be measured quickly, because the method requires that the bridge must be balanced several times.[5] See Fig. 10.

(f) Kelvin Double Bridge Method is described and discussed in IV.

(g) The Potentiometer Method is also described in V.

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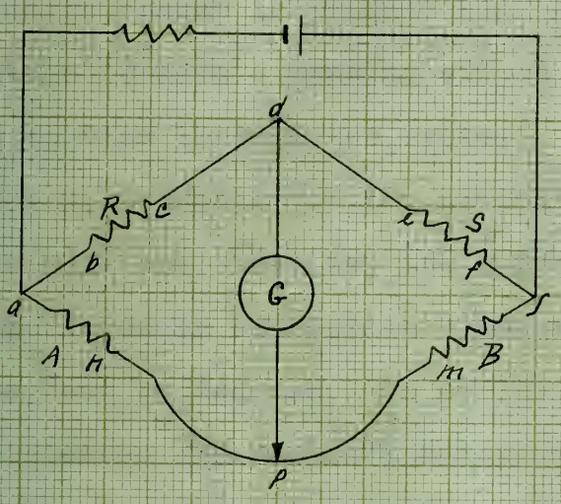
4 F. E. Smith "On the Method of High Precision for Comparison of Resistance", London Electrician, Vol. 57, pp. 876-978.

5 Albert Campbell "Measurement of Small Resistances", Phil. Mag., Vol. 6, pp 33-41, 1903.

Fig 11

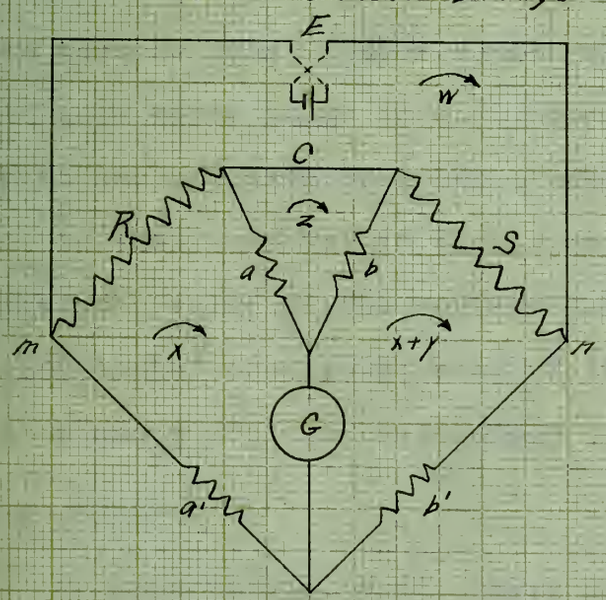
Wheatstone Bridge



R & S to be compared.
A arm = p to a
B arm = p to f.

Fig 12

Kelvin Double Bridge



$$\frac{R}{S} = \frac{a'}{b'} - \left\{\frac{c}{S} \times \frac{b}{a+b+c} \times \left(\frac{a'}{b'} - \frac{a}{b}\right)\right\}$$

III Description and Discussion of the Kelvin Double Bridge Method

Theory.- In the ordinary Wheatstones bridge a large current is passed for the purpose of increasing the sensitiveness of a galvanometer in the circuit. See Fig. 11.

The resistances a-d-e is very small compared to a-p-e. A current large enough to produce high sensibility but at the same time not too large to give a heating effect in the branch a-p-e, can be allowed to flow through the system. With the resistance of the galvanometer G, suitable for the circuit S and R may be compared under certain conditions; that is, the resistance of a-b, c-d, e-d, e-f, the current leads or connection wires must be known. Their value, of course, can be deduced. To evaluate them we employ the same device as Mathiessen and Hockin's bridge method.

The resistance standards, the shunts whose resistance is 1 ohm or less, are provided with a pair of drop lead wires besides the current terminal copper blocks. It is far easier to determine the value of drop lead copper wires than to find the resistance of current terminals made of heavy copper blocks or bars. Yet in the comparison of a resistance whose value is 0.001 ohm even 0.000001 of an ohm in series with it and not exactly known produces serious error.

In 1861 Lord Kelvin made a modification of Wheatstone's bridge so that the value of the current terminals of a resistance coil does not any longer occupy the arms of the bridge. See Fig. 12.[6]

R and S are the resistances of low value; c is a heavy copper

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6 Lord Kelvin, Proc. Royal Society, Vol. 9, 1861.

connection; a, b, a' and b' are composed of coils of rather high resistance compared to R and S. Hence the main current flows through S, C and R and thus the limit of current is the capacity of S and R. When no current flows through the galvanometer into Wheatstone's bridge (Fig. 11), the following relation holds true.

$$\frac{R}{S} = \frac{A}{B} \quad \text{or} \quad R = S\frac{A}{B}, \tag{1}$$

When the Kelvin double bridge is balanced, instead of (1) we shall have

$$\frac{R}{S} = \frac{a'}{b'} - \frac{c}{S} \cdot \frac{b}{a+b+c}\left(\frac{a'}{b'} - \frac{a}{b}\right) \tag{2}$$

In order to prove this formula let us consider the network (Fig. 12) again.

Let E = the voltage of the cell

B = the resistance of the battery arm MN

W = Imaginary current in MCNE circuit flowing in a clockwise direction

y = Imaginary current in RaGa' circuit

z = Imaginary current in Cab

x+y = Imaginary current in SbGb'

x+y-y = x = Imaginary current in G.

By Kirchoff's second law, we can equate

$$(B+R+C+S)\,w - S(x+y) - Ry - Cz = E \tag{3}$$

$$(G+b'+S+b)(x+y) - Gy - Sw - bz = 0 \tag{4}$$

$$(G+a'+R+a)y - G(x+y) - Rw - az = 0 \tag{5}$$

$$(a+b+c)z - b(x+y) - ay - Cw = 0 \tag{6}$$

Solving for x, the galvanometer current, we get by determinants

$$X = \frac{E \begin{vmatrix} (b'+S+b), & -b, & -S \\ (a'+R+a), & -a, & -R \\ -(a+b) & (a+b+C) & -C \end{vmatrix}}{D}$$

Where $D = (B+R+CS) \begin{vmatrix} (G+b'+S+b), & (b'+S+b), & -b \\ -G & , (a'+R+a), & -a \\ -b & , (\ -b \ ), & (a+b+c) \end{vmatrix}$

$$+ \begin{vmatrix} -S, & -(R+S), & -c \\ -G, & (a'+R+a), & -a \\ -b, & -b \ , & a+b+c \end{vmatrix} - R \begin{vmatrix} -S, & & \\ (G+b'+S+b), & & \\ -b, & & \end{vmatrix}$$

$$\begin{vmatrix} -(R+S), & -c \\ (b+b'+S), & -b \\ -b, & (a+b+c) \end{vmatrix} + C \begin{vmatrix} -S & -(R+S), & -c \\ (G+b'+S+b), & (b'+S+b), & -b \\ -G & (a'+R+a), & -a \end{vmatrix}$$

or

$$X = \frac{E \left\{ (a+b+c)(a'S-b'R) + C(a'b-b'a) \right\}}{D} \qquad (7)$$

When there is no current flowing through the galvanometer, X is zero, and that is true whatever the value of D. The right hand member is equal to zero.

When X = 0, we get

$$Rb' = a'S - \frac{C(a'b-ab')}{a+b+c}$$

or

$$\frac{R}{S} = \frac{a'}{b'} - \frac{C(a'b-b'a)}{(a+b+c)Sb'} \qquad (8)$$

Equation (8) can be reduced to

$$\frac{R}{S} = \frac{a'}{b'} - \frac{C}{S} \frac{b}{(a+b+c)} \left(\frac{a'}{b'} - \frac{a}{b}\right)$$

The term

$$\frac{C}{S} \cdot \frac{b}{a+b+c} \left(\frac{a'}{b'} - \frac{a}{b}\right)$$

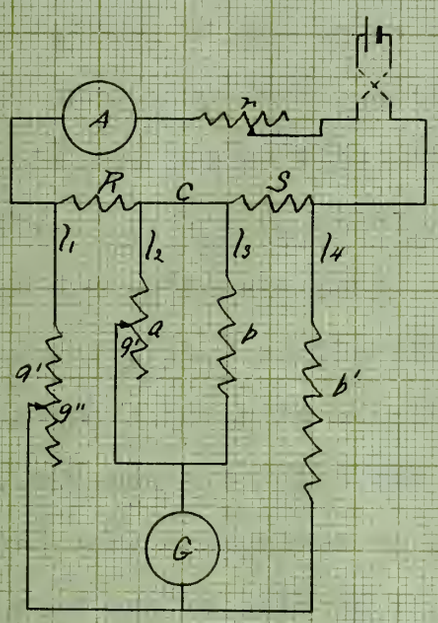
is called the correction term.

When $\frac{a'}{b'} = \frac{a}{b}$, the correction term becomes zero.

In practice, $\frac{a'}{b'}$ never equals $\frac{a}{b}$, however, the difference is very small

Fig. 13

Kelvin Double Bridge (Wolff)

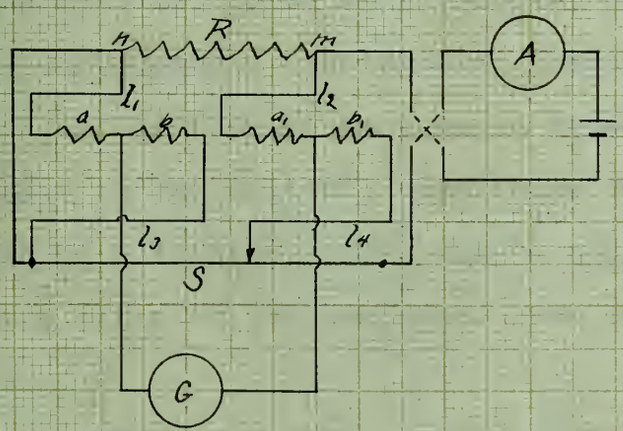


a & a' are dial type resistances
b & b, " plug " "

R & S to be compared

Fig. 14

Kelvin Double Bridge (Slide wire)



so that the correction is negligible in most cases.[7]

Types of Double Bridges on the Market.-

(a) First type.- Fig. 14. R is the resistance to be measured which is inserted between the two heavy binding posts M and N. The resistances a, b, a' and b' are three or four manganin coils of denominations ranging 10, 100, 1000, and sometimes 10000 ohms. It is possible for $\frac{a}{b} = \frac{a'}{b'}$, to be 10, 100 or 1000, while $\frac{b}{a} = \frac{b'}{a'}$ will be .1, .01, .001 or may be unity. l_1, l_2, l_3 and l_4 are flexible cables and each is a part of the four separate arms. S is a heavy uniform manganin wire or manganin strip used as the standard of comparison. A scale is provided along the slide wire S. This scale is usually graduated in such a way that any position of S can be read directly in ohms. This type of bridge is very convenient to measure the resistance of wire samples. The slide wire can be shunted if a large current is required. At the same time C must be shunted for it is permanently connected in the bridge. As soon as the slide wire S is shunted a new scale must be provided. Then, when this type of bridge is designed for several values of currents, several scales must be provided or several separate graduations must be marked on the scale, otherwise the size of S puts a limit to the current. The sliding contact is a part of one of the ratio arms so that poor contact would cause $\frac{a}{b}$ to be equal to or not equal to $\frac{a'}{b'}$. In the writer's experience the mere fact of pressing the slider down on S tight or loose made a difference in the reading as much as five parts in ten thousand, when ten ohm coils are used in the four arms.

(b) Second type.- Fig. 13. There is on the market another type of

7 Fleming "Laboratory Hand Book", Vol. I, pp. 274-275.
J. C. Maxwell "Theory of Electricity and Magnetism", Vol. 1, p. 488.

the double bridge. This type is composed of four arms, of which two are fixed and two are variable. The fixed arms made of two or three manganin coils are usually in the form of plug resistances. The resistances of these coils are commonly 25, 50, 100, 200. The variable arms are comprised of numbers of coils in series ranging .1 to 1000 or sometimes 10000 ohms. These coils are grouped in 9 x 0.1, 9 x 1, 9 x 10, 9 x 100, 9 x 1000, and sometimes 9 x 10000. These groups are put in decade dial form or plug resistance, and are so made that two arms can be varied at the same time and by the same amount. As shown in Fig. 13, a and a' are variable arms while b and b' are fixed arms.

These are only four ratio arms of the double bridge, the main part S and R connected by C are put together outside of the four given parts. S is connected to b and b' with lead wires and R is joined to a and a' by means of leads. In this type of bridge the amount of current is limited only by the capacity of R and S so that large currents can be used without serious heating effects when R and S are very small resistances.

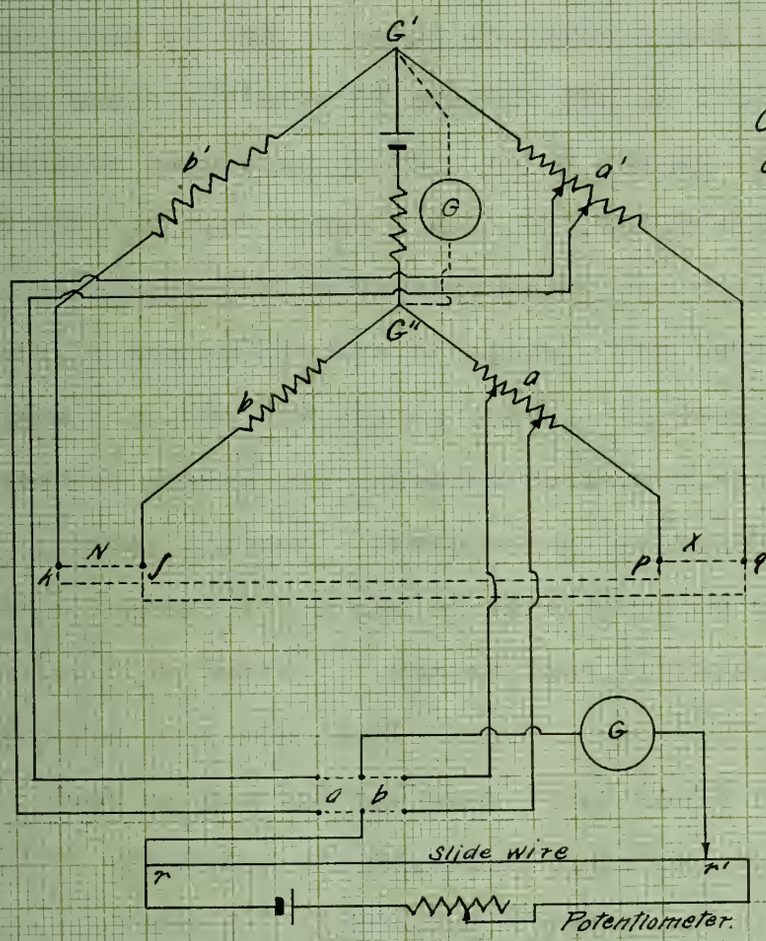
This form of bridge is manufactured by Leeds and Northrup Company, Philadelphia, and Otto Wolff Company, Berlin, in compact and convenient shape. In the use of these practical forms of the Kelvin double bridge, instead of formula (2), the formula

$$\frac{R}{S} = \frac{a}{b} = \frac{a'}{b'} \qquad (9)$$

is employed. Hence it is essential that $\frac{a}{b}$ should be equal to $\frac{a'}{b'}$, or the difference between them should be as small as possible.

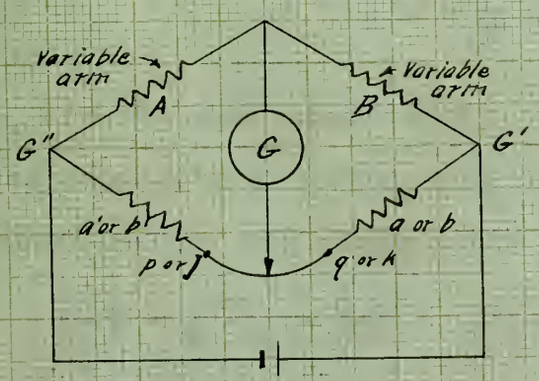
A bridge of this latter type which was made by the Otto Wolff Company and which was used by the writer for these experiments was tested for the interrelation of the four arms. In this bridge the

Fig. 15



Comparison of four arms of double bridge

Fig. 16



Comparison of arms by Wheatstone method.

Lettering, same as in Fig. 15.

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arms b and b' are composed of 25, 50, 100 ohms coil each. The arm a and a' are composed of 9 x .1, 9 x 1, 9 x 10, and 9 x 100ohm coils in dial form with slide contact in each. Every coil in the arm b was compared against the corresponding coil in the arm b'; and in the same manner all of the coils in the arms a and a' were compared with the coils whose resistances are one ohm or less by the potentiometer substitution method which will be explained later in this paper, Sec. 5. See Fig. 15 for connections. The coils whose values are more than one ohm were compared by Wheatstone bridge method, this bridge being especially set up for the occasion. The arms a and b in Fig. 16 are standard decade resistance boxes. A piece of uniform manganin wire of known resistance was inserted between P and Q or between L and K. The differences between corresponding individual coils or groups of coils in the arms a, a', b and b' were found on this wire. The results are tabulated in Table I. The maximum difference thus found among the coils was .016 per cent.

Let us now see how much error would result when formula (9) is used in the place of (2), when there may exist a difference of 0.02% between $\frac{a}{b}$ and $\frac{a'}{b'}$. Supposing S equal to .01 ohm, R equal to .01 ohm, a' equal to 100 ohms, b' equal to 99.98 ohms, a = b = 100 ohms, and c = .01 ohm. From (9) we get R

$$R = S\,\frac{a_1}{b} = S\frac{a_1'}{b'}$$

a_1' and a_1 are the same numerically, but differ from a and a' by the difference caused by b' being .02% off, and $\frac{a}{b}$ differs from $\frac{a'}{b'}$ by .02%. If the above values are substituted in (2)

$$\frac{0.01}{0.01} = \frac{100}{99.98} - \frac{0.01}{0.01} \cdot \frac{100}{200.01} \quad (1.00002 - 1.00) = 1$$

Without the correction term a, or a' would read 100.02 instead of 100.

Then it is necessary that the difference existing among the resistances of the coils in the four arms a', b', a, and b must be less than .01% when an accuracy of .02% is demanded. In the bridge made by the Otto Wolff Company any difference of more than .01% found among the coils can be adjusted by shunting such coil or coils, whereupon the bridge may be used relying on the direct readings without making corrections.

Precautions Involved in Use.- As it was stated in the previous section $\frac{a}{b}$ must be very nearly the same as $\frac{a'}{b'}$. Their difference must be less than .0001 from one another in all their values. Examining Fig. 13, l_1 is a part of a', l_2 is a part of a, l_3 is a part of b, l_4 is a part of b'. If these leads are each comparable to the four arms in resistance they should be taken care of. When $a_1 = a' = b = b'$, the condition imposed upon the four leads is that $l_1 = l_2 = l_3 = l_4$, regardless of their actual value. See Table II. If however, as soon as a and a' are n times b and b', the value of the four leads must be added to the resistance arm to which each is connected. To illustrate let a = a',= b = b' = 100 ohms, $l_1 = l_2 = .01$ ohm, $l_3 = l_4 = .05$ ohm. Using formula (9)

$$\frac{R}{S} = \frac{a_1'}{b'} = \frac{a_1}{b}$$

Then

$$\frac{R}{S} = \frac{a + l_2}{b + l_3} = \frac{a' + l_1}{b' + l_4} = \frac{100.01}{100.05}$$

Therefore $\frac{a_1'}{b'} = \frac{a_1}{b}$ = less than 1, while $\frac{R}{S} = 1$. The error in such a case is .04%. Let us take another case when R = 2S and when all the leads are the same in resistance.

$$a = a' = 200 \text{ ohms} \qquad b = b' = 100 \text{ ohms}$$

$$l_1 = l_2 = l_3 = l_4 = .02 \text{ ohm}$$

$$\frac{a + l_2}{b + l_3} = \frac{200.02}{100.02} \quad \text{and} \quad \frac{a' + l_1}{b' + l_4} = \frac{200.02}{100.02}$$

and not $\frac{200}{100}$ or $\frac{200.02}{100.01} = 2$. The error produced in this case is .01%. See Table II for actual experiments.

In this table it can be seen that when R is nearly the same as S and one of the four leads has several times the resistance of the others, the error is much greater. (See last two instances in Table II.)

$$a = b = b' = a' = 100 \text{ ohms}$$

$$l_1 = l_2 = l_3 = 0.01 \quad \text{and } l_4 = 0.05 \text{ ohms}$$

$$\frac{a_1 + l_1}{b_1 + l_4} \neq \frac{a + l_2}{b + l_3} \quad \text{or} \quad \frac{100.01}{100.05} \neq \frac{100.01}{100.01}$$

the error is nearly .04%.

When the double bridge method is used, from the above illustration and from actual measurements taken, the value of the four leads must be known within a few per cent of their actual value even when their resistances are low. If it is expected to make measurements within .02%. In the precision measurement of high grade the potential leads and any other leads which occupy the places of l_1, l_2, l_3 and l_4 are measured for any particular set up.[8] This way of evaluating the resistance of the leads is too slow and often is not possible to be worked out because of peculiar construction of some shunts. It is the practice to make corrections upon the readings from the approximate values of the leads and the correct values of the coils in a, a',

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8 E. B. Rosa "Measurements of Small Resistances", Bureau of Standards Bulletin, Vol. 5, pp 413-434, 1909.
C. V. Drysdale "Comparison of Resistances", London Electrician Vol. 60, pp 467-566, 1908.
"On the Method of High Precision for the Comparison of Resistances", London Electrician, Vol. 57, p. 1010.

Fig. 17

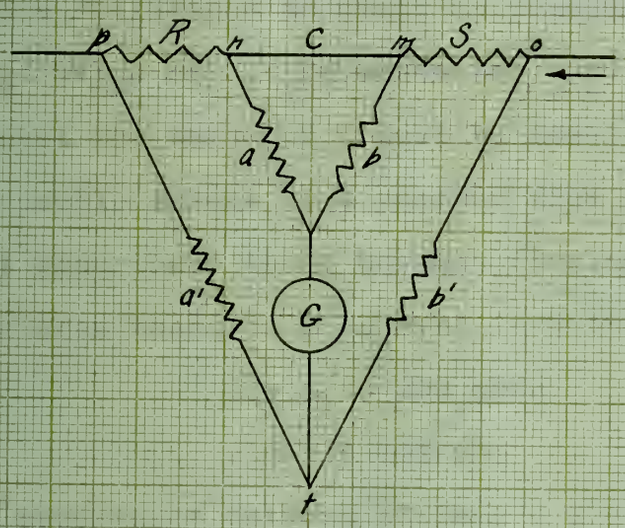
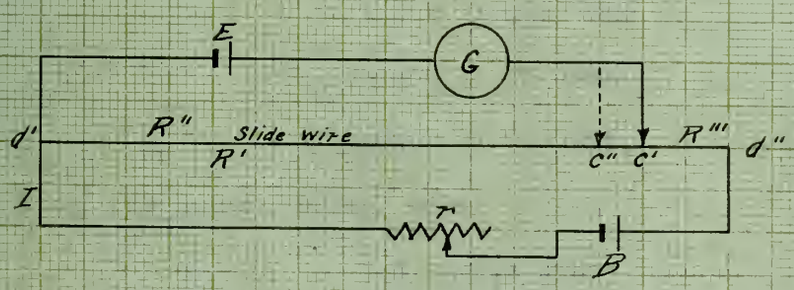


Fig 18.

Potentiometer Principle.



$R = d'd''$
$R' = d'c'$
$R'' = d'c''$
$R''' = c'd''$

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b and b' arms.

Inspecting the correction terms in the formula (2) besides the factor $(\frac{a}{b} - \frac{a'}{b})$ there are the factors $\frac{c}{S}$ and $\frac{b}{a+b+c}$. In order to minimize error caused by inequality existing between $\frac{a}{b}$ and $\frac{a'}{b'}$ it is obvious that C must be smaller than S. For the connection C use a flexible copper cable which has a good flat compact surface at its ends, with a resistance as small as possible. The errors magnified by a large value of C, the connector, are shown in Table III. In another respect the value of C must be small. Let us consider the network of the double bridge from another view point. See Fig. 17.[9] It is seen that when there is no current flowing through G, a+b is shunted by C. Even when there is a current in G this shunting effect exists.

Let $a = a_1 = 200$ ohms $\quad b = b' = 100$ ohms

$c = S = .01$ ohm $\quad R = .02$ ohm

The resistance between bO is $R + S + \frac{c(a+b)}{c+a+b}$ in one branch and a' + b' in the other.

$$nm = \frac{c(a+b)}{c+a+b} = 0.3 \text{ ohm}$$

$$pnmo = 0.33 \text{ ohm}$$

$$o+b = 300 \text{ ohms}$$

When

$$C = 0.001 \text{ ohm}$$

$$nm = 0.03 \text{ ohm}$$

and

$$pnmo = 0.06 \text{ ohm},$$

while on the other hand, when

$$C = 0.1 \text{ ohm}$$

$$nm = 3 \text{ ohms} \quad \text{nearly}$$

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9 J. H. Dellinger and F. A. Wolff "Temperature Coefficients of Copper", Phys. Rev., Vol. 33, p. 215-216, 1911.

and

$$pnmo = 303 \text{ ohms}$$

When C is of much smaller value than S or R, mn is a very small part of pnmo. When C is greater than either of the two the resistance mn plays a prominent part in pnmo. When C is comparatively small, any slight deviation in the resistance of the a and b arms has a negligible effect upon the value of nm. Thus it is evident that the resistance of a' and b' must be known exactly and the formula

$$\frac{R}{S} = \frac{a'}{b'}$$

should be used for all practical purposes. When several sliding contacts are used in the variable arms of the bridge; namely, a and a', the contacts of the slider must always be kept clean, otherwise often variations of ± 0.005 ohm will result in the readings.

When R is at a temperature above or below room temperature, as in the case of the temperature coefficient tests of various metals and alloys, or when a too heavy current is flowing through R and S one must watch out for thermoelectromotive forces. The errors arising from thermoelectromotive forces can be avoided either by taking two readings, reversing the current, or by closing the galvanometer key before the battery key.[10] In either case the observer must note the thermoelectromotive force before and after the readings are taken.

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10 W. A. Price, "Methods of Measurement of Resistance".

V The Potentiometer Method

Description and Theory.- A potentiometer is a long uniform wire in an electrical circuit. The whole wire or a portion of it represents certain voltages under certain conditions. The general principle or theory is as follows. Fig. 18 represents in outline the simple potentiometer for measuring voltages not exceeding that of the auxiliary battery. A battery B sends a current through a resistance $R + r = R' + R''' + r$, in which r may be varied to keep the current constant, as the e.m.f. of B changes. E is another source of current, usually a standard cell. The current from B is opposing that from E. The point C is connected on R at such place that the e.m.f. of E is balanced by the fall of potential in that portion of R or R'. This condition is indicated when there is no deflection of the galvanometer. By Ohm's law we have

$$E = \frac{R'}{R+r}e = \frac{R'}{R'+R'''+r}e \qquad (1)$$

where e is e.m.f. of B of the battery B.

Since the current

$$I = \frac{e}{R+r}$$

we may put (1) as $E = R'I$ which is the potential drop from d' to d" on the uniform wire. If E is replaced by any other source of current E', and at C' the galvanometer gives no deflection

$$E' = \frac{R''}{R+r}e = I'R'' \qquad (3)$$

which is the potential drop from d' to c". Dividing (3) by (2) we have

$$\frac{E'}{E} = \frac{I'R''}{IR'} \qquad (4)$$

If in securing the two balances $\frac{e}{R+r}$ remains unchanged, $I = I'$ and (4) becomes

$$E' = E\frac{R''}{R'} \qquad (5)$$

Then it is seen that the relation of one voltage E to the other E' is the same as the resistance in the one case to that in the other; or, in other words, voltage is proportional to the resistance on the wire.

Instead of E' of a battery a potential drop across a part of a conductor in some other electrical circuit is taken, and that is made equal to E', C' being the balancing point.

$$E' = I_1R_1 = IR'' \tag{6}$$

where I_1 is the current in the secondary circuit in which R_1 is located. Since

$$E' = E\frac{R''}{R'}$$

$$I_1R_1 = E\frac{R''}{R'}$$

$$R_1 = E\frac{R''}{R'I_1} \tag{7}$$

If the value of R_1 is known, R_1 can be found. There are two methods of finding the value of resistance corresponding to R_1 using the potentiometer.

(1) When a standard cell is used as a basis of comparison. See Fig. 19. In this figure ab is a double pole double throw switch. Let

R_1 = the resistance to be measured

R = the resistance of the slide wire and coils

E = the voltage of a standard cell

S = the resistance of a uniform wire

R' = the resistance of the coils connected in series with S.

When E is connected to G by closing the switch A and a balance is obtained at C' by varying r'

$$E = (R' + S')I' \tag{8}$$

Now A is opened and b is closed and a balance is again made by moving

C' to C"

$$I''R_1 = I_1'(R' + S'') \tag{9}$$

Dividing (9) by (8) gives

$$\frac{I''R_1}{E} = \frac{I_1'(R' + S'')}{I'(R' + S')} \tag{10}$$

When $I' = I_1'$, (10) becomes

$$R_1 = \frac{E}{I''} \frac{R' + S''}{R' + S'}$$

Dividing R which is equal to R' + S into small equal parts and calling each part r

$$R = nr$$

where n is the number of divisions. Then

$$E = n'rI'$$

where n' is some convenient integer; that is, n' is numerically equal to the voltage E, and

$$I''R_1 = (N''r)I'$$

Since n" is also numerically equal to the potential drop d'C"

$$R_1I'' = E\frac{n''}{n'}$$

$$K = \frac{E}{n'}$$

Since n' is numerically equal to E

$$K = 1$$

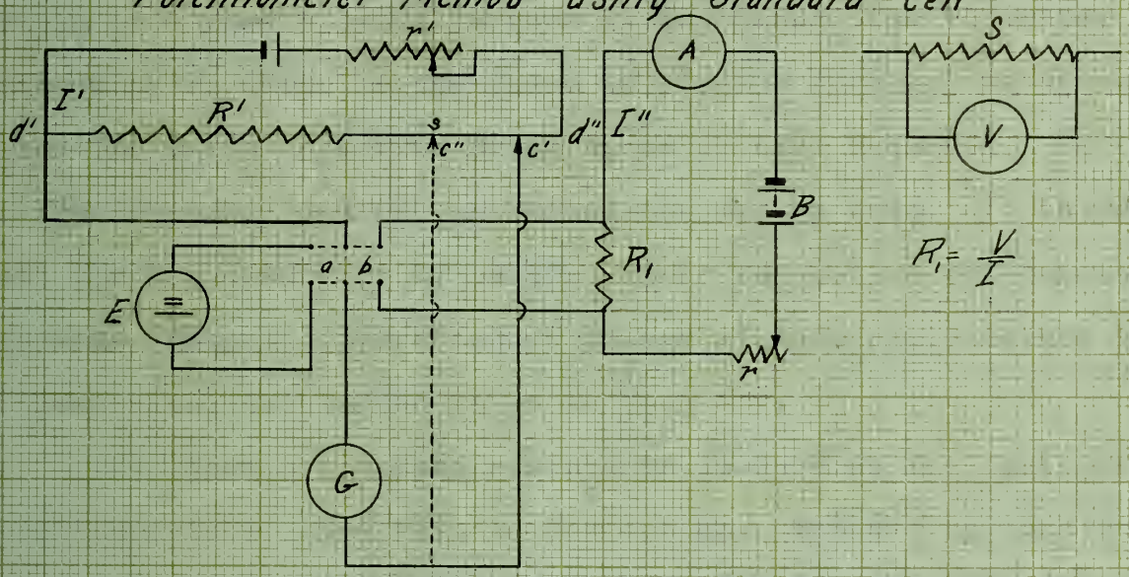
$$R_1I'' = n''$$

$$R_1 = \frac{n''}{I''} \tag{11}$$

Suppose that n'r and n"r, or simply n' and n", can be read in five digits on a scale. Then a difference of one part in 100,000 can be observed directly. If I" is read on a standard ammeter which indicates a difference of .001 ampere at best, then the method falls short

Fig. 19.

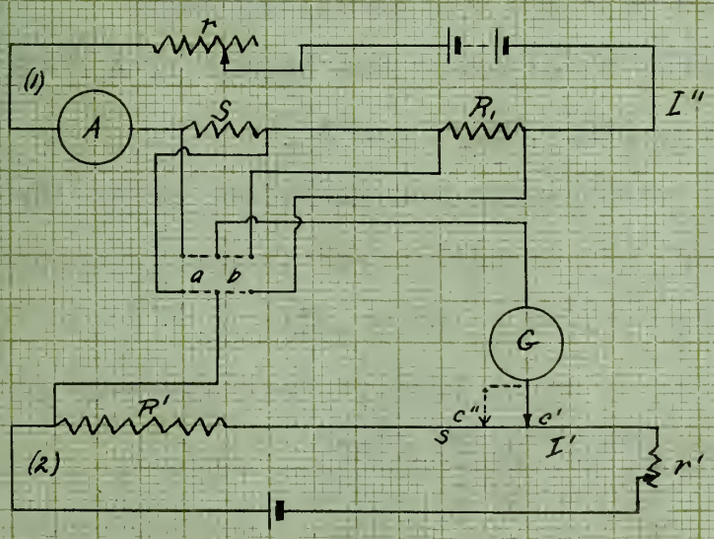
Potentiometer Method using Standard Cell



$R_1 = \frac{V}{I}$

Fig. 20.

Potentiometer Method using Standard Resistance.



$R_1 = S n''$

of the requirement of detecting a difference of .0002. Instead of using an ammeter a voltmeter over a standard resistance S which is inserted in place of the ammeter may be used to indicate the current I" indirectly, as

$$I'' = \frac{V}{S}$$

But in this case a small current flows through the voltmeter. This introduces an error in the value of I", and further, no standard voltmeter indicates such fine differences as here required. In the place of the voltmeter another potentiometer may be used. It is very difficult for two observers to balance their respective potentiometers at the same instant. Therefore, although it is a rapid method this way of measuring resistances fails in accuracy.

(2) When a standard resistance is used as the basis of resistance comparison. See Fig. 19 for set up. The set up consists of two distinct circuits, one is a potentiometer, and the other is a circuit in which the standard of resistance and the resistance to be measured are inserted and also in which a rather heavy current is flowing. Let

S = the standard resistance

R_1 = the resistance to be determined

Other notations are the same as before.

When A is closed and there is no deflection in the galvanometer

$$I''S = I'(R' + S') = n' rI' \qquad (12)$$

The double pole switch is now thrown over to b, and the potentiometer is balanced by moving the slider to C". Then

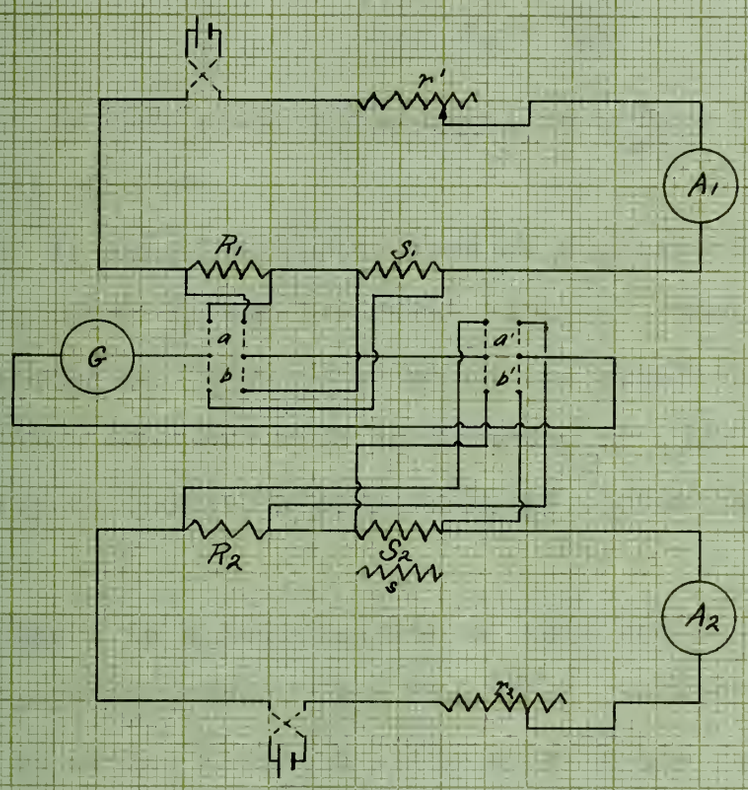
$$I_1''R_1 = (R' + S'')I_1' = n'' rI_1' \qquad (13)$$

Dividing equation (13) by (12) gives

$$\frac{I_1''R'}{I''S} = \frac{I'(R' + S'')}{I'(R' + S')} = \frac{I_1'n''r}{I'n'r} \qquad (14)$$

Fig. 21

Shunt Potentiometer Method



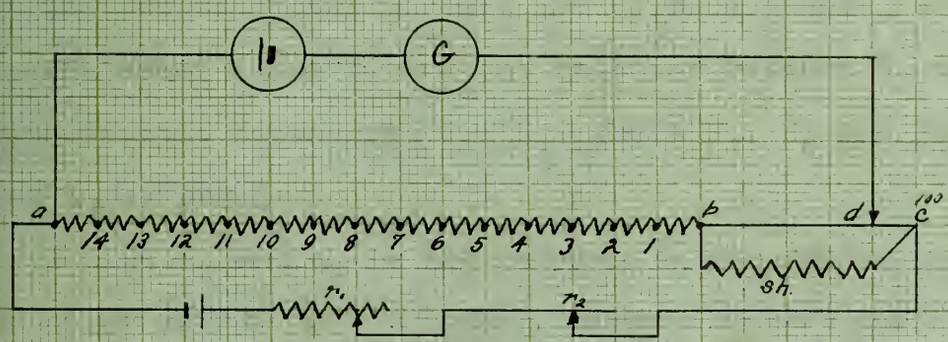
$R_1 = 10\,S_1$
$R_2 = 10\,R_1$
$S_2' = 10\,R_2$

$$\frac{R_1}{S_1} = \frac{R_2}{S_2'}$$

S_2' = shunted value of S_2

Fig. 22

Potentiometer Coils & Slide Wire



ab = 15 x 1 ohm
bc = 1 ohm

The currents in the two circuits remaining unchanged while the two balances are made, we have $I'' = I_1''$, and $I' = I_1'$, and equation (14) becomes

$$R_1 = S\frac{n''}{n'} \tag{15}$$

When n' is made numerically equal to S $\frac{S}{N'} = 1$ and equation (15) reduces to $R_1 = n''$ (16)

When n' is numerically the same as S except for the position of the decimal point $\frac{S}{n'} = K$ determines that n" should be in some decimal multiple of S. This is known as the substitution method. When the value of S is known with an accuracy of .00001 and n' or n" is read directly to .00001, a difference of .00002 can be found without difficulty.

There is a modification of this second method known as a shunt potentiometer method. (See Fig. 21) Instead of using a number of coils in series with the slide wire in the potentiometer circuit, two standard resistances are used in the other circuit, and another standard resistance is connected in series with the resistance whose value is to be determined. Instead of using the slider one or two standards are shunted in order to obtain a balance. The two resistance standards in the potentiometer circuit are of much higher resistance than those in the other circuit, and the relation between R_2 and S_2 is nearly the same as that between R_1 and S_1, where S_1, S_2 and R_2 are the standards and R_1 is unknown.

Let A_1 and A_2 be the current in the two circuits. In the figure a, b, a' and b' are switches. When a and a' are closed the potential drop in R_1 is opposing that in R_2, and when b and b' are closed S_1 and S_2 are opposing each other. When a and a' are closed r_2 the rheostat

is regulated until there is no deflection in the galvanometer. Then we get

$$R_1A_1 = R_2A_2 \tag{17}$$

Now b and b' are closed, and again a balance is made by shunting S_1 or S_2. We then have

$$S_1A_1 = S_2'A_2 \tag{18}$$

provided that A_1 and A_2 remain unchanged.

Dividing (18) by (17) gives

$$\frac{R_1A_1}{S_1A_1} = \frac{R_2A_2}{S_2'A_2}$$

or

$$R_1 = S_1\frac{R_2}{S_2'}$$

If the value of the resistances used to shunt S_1 or S_2 can be read directly in ohms to five figures, the method is well adapted for unity or .1 ratio between R_1 and S_1.

Types of Potentiometers on the Market.- There are two kinds of potentiometers manufactured by commercial establishments.

(q) The potentiometer wire is divided into fifteen large equal parts. Of these fourteen parts are composed of coil resistances, while the fifteenth is made of a uniform wire. See Fig. 22. The resistance of each coil and slide is made equal within .0001. This individual resistance ranges from a fraction of an ohm to several ohms. Contact over the fouteen coils is made by a sliding finger, and over the slide wire by a slider. The slide wire may be stretched straight or is wound on a cylinder. It is divided into 1000 or 10000 equal parts or subdivisions. The points a and d are the moveable finger and slider respectively. The current is adjusted by varying R_1 the coarse, and R_2 the fine adjustable rheostats.

Fig 23

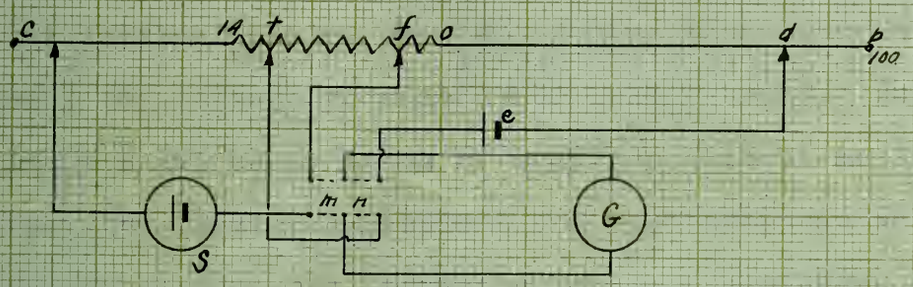
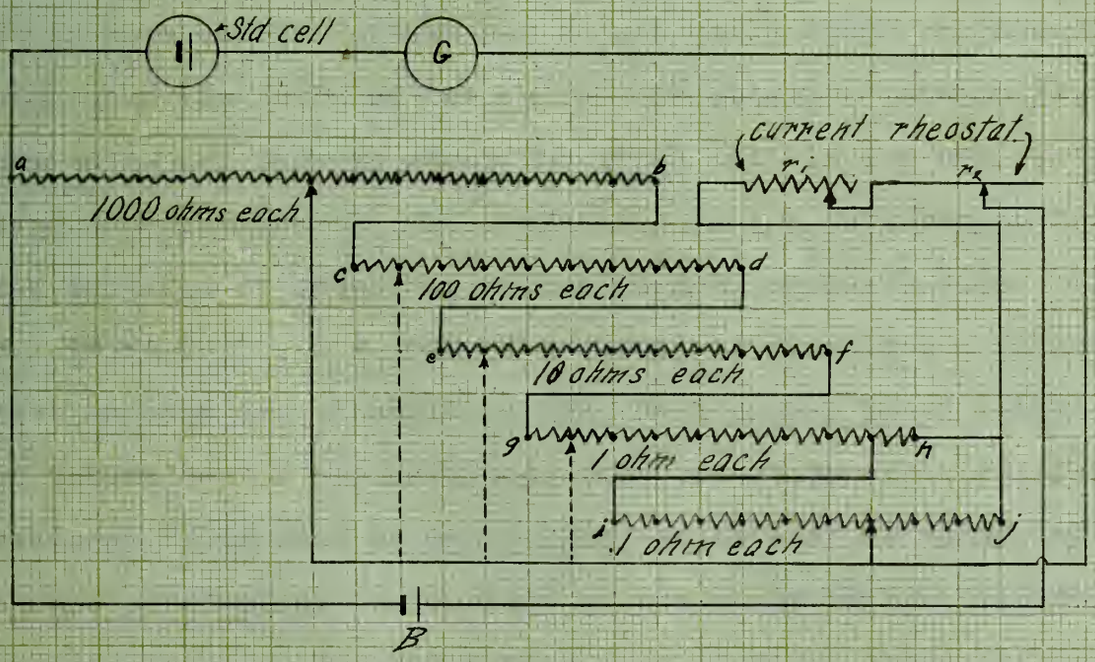


Fig 24.

Construction of Potentiometer Coils.

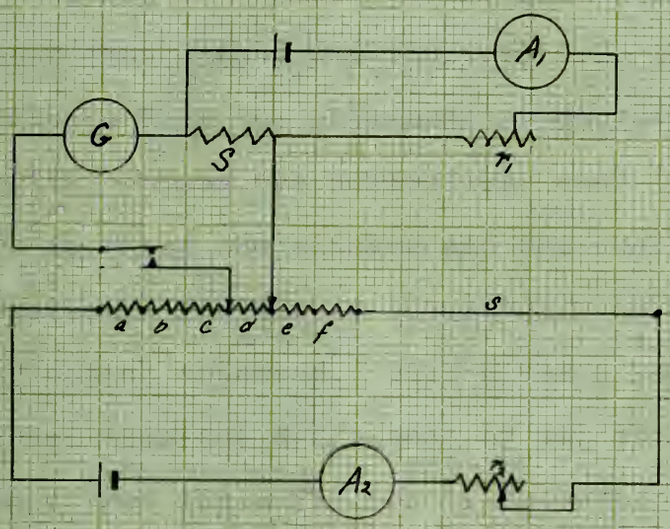


There is another type of this kind of potentiometer which is provided with two slide wires, one on each side of the coils, and three moveable contact makers, c, t, and d, and one fixed, f, in Fig. 23. The setting f to c is made according to the value of a standard cell, this setting remains constant during the measurement. By sliding t and d the voltage of some other cell is read. In this type the setting can be examined by means of a switch without moving the slider each time. It is very convenient and good speed can be obtained, either in comparison of e.m.f, of batteries or resistances. This type is made by Leeds and Northrup Company, Philadelphia, and Robert Paul, Cambridge England, while the preceding type is made by many manufacturers.

(2) In the second type of potentiometer there is no slide wire, but there are a number of coils made of manganin wire connected in series. These coils are so constructed and adjusted that any one of the coils corresponds to some definite part of a long uniform wire. See Fig. 24. There is no coil whose resistance is less than one ohm. It is much easier to make and adjust coils whose resistance is rather high than the coils of low value. To obtain .1 ohm coils, nine one ohm coils are made to shunt the tenth coil in one ohm series. In this type of potentiometer the resistance of the coil ranges from one to ten thousand or sometimes to one hundred thousand ohms. Viz. 9 x 1, 9 x 10, 9 x 100, 9 x 1000, 9 x 10000, or sometimes 9 x 100000. Contact on each set of coils is made by a copper brush slider. This type of potentiometer has advantage over the slide wire type for several reasons; it is easy to manipulate, it is easy to read, and there is no chance of scraping the surface of the slide wire. Otto Wolff Company, Berlin, Germany, furnishes this type of first class potentiometer. However, when checking the value of coils or slide wires necessary, the

Fig. 25

Method of Checking Coils & Slide Wire



S = about ½ ohm

first type is much simpler in calibration and adjustment than the high resistance coil type. When there arises any doubt that the resistance of coils and slide wires is not equal, these values must be compared one with another. Make connections as shown in Fig. 25. Set up a circuit S, the manganin coil, whose resistance is about the same as any one of the potentiometer coils. Take any one of the fourteen coils as a basis of comparison. Adjust current in A till the potential drop over the coil thus chosen and the coil S is the same. Then move two sliding contacts from the galvanometer circuit upon the rest of the fourteen coils, noting the deflection each time. Then the slide wire is put in these two sliders and the current adjusted till there is no deflection. Now one of the sliders is moved a known value on S and the deflection observed. This gives a value to each millimeter deflection and thus the difference of all the coils and the slide wire can be found. See Table XII.for illustrations.

Precautions Involved in the Measurements.- Before any potentiometer is used a normal current must be flowing through it for some time. The reason for this is that the conductors in the whole circuit must be at one temperature and that its temperature must not be changed during measurements, and that when a storage battery circuit is just closed there is a little variation in the voltage of the batteries for the first several minutes. For the same reason the current in the auxiliary circuit must be flowing for some time. All the conductors in the auxiliary circuit must be large enough so that they are free from heating effects, otherwise a cold blast of air upon a hot wire will cause a shifting of the current. In the potentiometer methods it is assumed that the currents in the two circuits remain the same, and that the error is proportional to the change in the current. Therefore,

storage cell S used in the potentiometer or in the other circuit must be watched closely so that the current will be uniform. In using a potentiometer sometimes n' is reproduced after n" reading is made by again adjusting A_2 current in the resistance measurement. There is another way of taking care of the drift of current; namely, after a potentiometer is once set up for the value of S, several readings on S and R are taken without adjusting the current. From these readings average values of n' and n" are found from the formula

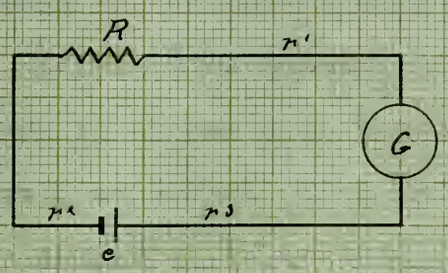
$$R_1 = S\frac{n''(\text{average})}{n'(\text{average})}$$

Such manipulation is necessary when a heavy current is used in the auxiliary circuit. The two circuits must be well insulated from each other and from the ground, otherwise false deflections of the galvanometer will occur. For the purpose of eliminating errors arising from the change in the current, several readings must be made for each measurement. If the variation among the readings is too large it is advisable to wait until the currents settle down. Sometimes it becomes necessary to change the batteries. The time interval between setting of the potentiometer and reading on it must be as small as possible.

VI Selection of Galvanometer

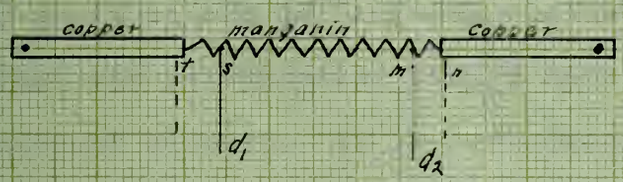
When accuracy is required in electrical measurements, besides the correctness of the value of the standards and a flexibility of the type of instruments used, sensitiveness of detectors is essential. Under the given condition in a bridge circuit sensitiveness depends upon the galvanometer. Rapidity also depends upon how quickly the galvanometer responds, and indicated a change of current. Sensitiveness of a galvanometer is proportional to the length of the coil wire and also to the square root of the resistance of the coil. Rapidity is also proportional to the period of the galvanometer. An underdamped

Fig. 26



Construction of Resistance Standards.

Fig 27

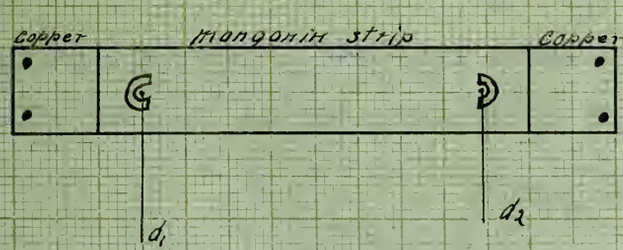


mn ≠ rs in resistance.

d_1 & d_2 = potential leads.

Fig. 28

Fig. 29.



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moving coil type of galvanometer which is made by a long fine wire has a long period. Such a galvanometer is very sensitive. On the other hand, overdamped galvanometers have a short period, but are not sensitive enough. Either time element or sensitiveness must be sacrificed when one wishes to choose a galvanometer for measurements which require accuracy as well as speed. If a galvanometer of desired sensibility is critically damped it is most satisfactory. In the measurement of shunts and other low resistance materials the desirable period for a galvanometer ranges from four to eight seconds, and the suitable sensibility of it is 9×10^{-8} amperes per millimeter deflection. The resistance that the galvanometer should have depends upon the resistance of the external circuit. It is convenient to have a formula which expresses the relation between the resistance of the galvanometer and the given external circuit. Let us consider a circuit in which a galvanometer, a resistance, and a source of electromotive force are connected in series. See Fig. 26. R is the external resistance, and r_1, r_2, and r_3 are the resistances of the connecting wires, and R_b the resistance of the battery. The total resistance in the circuit is

$$R + G + R_b + r_1 + r_2 + r_3,$$

but r_1, r_2, r_3 and R_b are very small compared to R and G.

The current in the circuit is then

$$i = \frac{e}{R + G}$$

Let R be changed by dR and i will be also changed by di. The smallest current that can be detected by the deflection of the galvanometer is di_g due to change idR.

$$di_g = \frac{idR}{R+G} \qquad (1)$$

in this particular case $di_g = di$. Therefore,

$$\frac{di}{i} = \frac{dR}{R+G} \quad (2)$$ [11]

With a given resistance and galvanometer, the ratio of the smallest change in resistance which can be detected, to the sum of the given resistances, is equal to the ratio of the smallest current which can be detected by the galvanometer deflection to the total current thru the circuit. When G = R

$$\frac{di}{i} = \frac{dR}{2R} \quad (3)$$

Since the deflection of the galvanometer is proportional to the length of the coil, to the current through it, and to the square root of its resistance, we may write [12]

$$D \quad di\sqrt{G}$$

$$KD = di\sqrt{G} \quad (4)$$

$$di = \frac{KD}{G} \quad (5)$$

From (3) and (5)

$$\frac{dR}{R} = \frac{2KD}{i\sqrt{G}} \quad (6)$$

or

$$dR = \frac{2R}{\sqrt{G}} \cdot \frac{KD}{i}$$

Let us apply this principle in the case of a type of Kelvin double bridge. In referring back to Fig. 12

Let i = the current through R

i = the current through S

i-i(a+b) = the current through C.

The balance of the bridge was disturbed by the change in R - dR. Then R + dR is the changed resistance, and the current is changed from i to

11 Arthur Schuster "On the Measurements of Resistance", Phil. Mag., Vol. 39, p. 175.

12 Professor A. Gray "Absolute Measurements in Electricity and Magnetism, Vol. II.

O. Heaviside "Formula for Sensitiveness of Galvanometer", Phil. Mag., Vol. 45, p. 114, 1873.

i - di. The total change in the potential drop is that branch of the bridge is then

$$(R + dR)(i - di) - Ri = idR - Rdi - dR\,di$$

the last term dR di is extremely small. The change in potentiometer over the S branch will be

$$S(i - di) - Si = Sdi$$

We can also assume that this little change of potential in R does not alter the current from the battery. This condition is the same as though the battery is cut off. When the bridge is balanced

$$\frac{R}{S} = \frac{a}{b} = \frac{a'}{b'} \qquad (1)$$

In practice a = a" and b = b' (2)

When the galvanometer is closed after this change is made, a current flows through it which is due to the e.m.f., (idR - R di) and the e.m.f., Sdi. Also a current flowing through S due to this change is equal to -S R di/(R+S)(r+G),

$$r = \frac{ab}{a+b} + \frac{(R + a')(S + b')}{R + S + a' + b'}, \qquad (3)$$ [13]

and lastly a current flowing through R due to e.m.f. S di is +S R di/(R+S)(r+G). Now equations (3) and (4) are the same, but their signs are opposing. Hence they cancel each other.

The other e.m.f. which also causes a current through the galvanometer is idR. The current due to this e.m.f. in the whole bridge circuit will be

$$i' = \frac{idR}{G + \frac{ab}{a+b} + \frac{(a'+R)(b'+S)}{R+S+a'+b'}} \qquad (5)$$

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13 Mr. F. E. Smith, "On the Method of High Precision for the Comparison of Resistances", London Electrician, Vol. 57, pp 976-978.

The current that flows through the galvanometer is then

$$i_g = i' \frac{S + b'}{S+R+a'+b'} \tag{6}$$

$$i_g = \frac{S + b'}{S+R+a'+b'} \cdot \frac{idR}{G + \frac{ab}{a+b} + \frac{(a'+R)(b'+S)}{R+S+a'+b'}} \tag{7}$$

i_g is a maximum when G equals the resistance of the external circuit, r, or

$$G = \frac{ab}{a+b} + \frac{(a'+R)(b'+S)}{R+S+a'+b'} \tag{8}$$

Now the sensitiveness of the galvanometer varies as the square root of its resistance, hence the deflection of the galvanometer is proportional to

$$\frac{\sqrt{G}\ idR}{G + r} \cdot \frac{b' + S}{S+R+a'+b'} \tag{9}$$

From (8) and (9) we get sensitiveness proportional to

$$\frac{\sqrt{G}\ idR}{2G} \cdot \frac{S + b'}{R+S+a'+b'} \tag{10}$$

or

$$KD = \frac{idR\sqrt{G}}{2G} \cdot \frac{S + b'}{R+S+a'+b'} \tag{11}$$

or

$$\frac{i_g}{i} = \frac{dR\sqrt{G}}{2G} \cdot \frac{S + b'}{R+S+a'+b'}$$

$$KD = \frac{idR}{2} \sqrt{\frac{(S + b')^2}{\frac{(R+S+a'+b')}{G}}} \tag{12}$$

$$KD = \frac{idR}{2} \sqrt{\frac{(S+b')^2}{(R+S+a'+b')^2} \Big/ \frac{(R+a)(S+b)}{R+S+a'+b'} + \frac{ab}{a+b}} \tag{13}$$

Equation (13) can be reduced to

$$KD = \frac{idR}{2} \sqrt{\frac{(a+b)(2a+R)}{Rb}} \tag{14}$$

The ideal resistance of the galvanometer by (8) is

$$G = \frac{ab}{a+b} + \frac{(R+a')(S+b')}{R+S+a'+b'}$$

Since R and S are comparatively small and a' = a, and b = b', we have

$$G = 2\frac{ab}{a+b} \tag{15}$$

Let the best galvanometer resistance be g for any given circuit, and ng for an actual galvanometer resistance. The deflection is proportional to

[14]

$$\frac{\sqrt{ng}}{(n+1)\sqrt{g}} \tag{16}$$

When n = 1 the deflection is the maximum. The deflection for any given galvanometer resistance caused by a change dR is proportional to the current caused by

$$iR + idR \text{ a } Rdi - Ri = idR - Rdi$$

Supposing change di is very small compared to i. e.m.f. change in R is idR. Current through a' due to this e.m.f. is

$$I_a = \frac{idR}{a'+a+g\frac{(b+b')}{g+b+b'}} \tag{17}$$

and the current through the galvanometer is

$$I_g = i_a\frac{b+b'}{b+b'+g} = \frac{idR(b+b')}{(a+a')(g+b+b')+g(b+b')} \tag{18}$$

$$\frac{DK}{i} = \frac{dR(b+b')}{(a+a')(b+b'+g)+g(b+b')} \tag{19}$$

Where D is the deflection and K is the galvanometer constant.

When a = a', b = b'.

$$\frac{DK}{i} = \frac{dR\ 2b}{4a(b+g)+g\ 2b} \tag{20}$$

From equation (6) we have

[15]

$$\frac{dR}{R} = \frac{DK}{iR}\ \ 2a+g(1+\frac{a}{b}) \tag{21}$$

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14 Arthur Schuster, "In the Measurement of Resistance", Phil. Mag. Vol. 39, p. 175.
Mr. F. E. Smith, "On the High Precision For the Comparison of Resistance", London Electrician, Vol. 57, p. 976.

15 E. F. Northrup, "Methods of Measurement of Electrical Resistance"

Equations (14) and (21) show the galvanometer sensitiveness due to a small change in one of the bridge arms. Equation (21) is more fitting than (14) in practice.

Selecting a galvanometer in potentiometer work involves the same condition as that for bridge work. However, potentiometer work requires a galvanometer which is sensitive to a very small change of its terminal voltage, and must be well advanced so that the deflection can follow the change of voltage due to change in current, or in resistance of the two circuits.

Let us consider what should be the resistance of the galvanometer in any given condition of the two circuits. See Fig. 20.

Total resistance in (1) = $R_1 + S + r_1$

Total resistance in (2) = $R' + r_2$

A small change in R_1, call it dR_1. The potential drop change over R_1 due to dR_1 is

$$(R_1+dR_1)(I''-di) - R_1I'' = I''dR_1 - R_1di - dR_1di \qquad (1)$$

The term $-dR_1di$ is extremely small, and since R_1 is a small quantity R_1di must be also very small. Neglecting these two terms, we are going to consider the term $I''dR_1$. When the system is balanced, this small change does not effect the current in the system other than the galvanometer circuit. The resistance of the galvanometer circuit is

$$G + \frac{R_1(S+r_1)}{R_1+S+r_1} + \frac{R'r_2}{R'+r_2} \qquad (2)$$

The current in the galvanometer due to the e.m.f. $I''dR_1$ is

$$i = \frac{I''dR_1}{G + \frac{R'r_2}{R'+r_2} + \frac{R_1(S+r_1)}{R_1+S+r_1}} \qquad (3)$$

The best resistance of the galvanometer, to make i a maximum, is

$$G = \frac{R'r_2}{R'+r_2} + \frac{R_1(S+r_1)}{R_1+S+r_1}$$

Then the sensitiveness is proportional to [16]

$$\frac{I''dR_1}{2R_1} \cdot \frac{\sqrt{R_1}}{\sqrt{\frac{R'r_2}{R'+r_2} + \frac{R_1(S+r_1)}{R_1+S+r_1}}} \tag{4}$$

S, R' and R_1 are very small compared to r_1 or r_2. Hence (4) becomes

$$\frac{I''dR_1}{2R_1} \cdot \frac{\sqrt{R_1}}{\sqrt{R_1+R'}} \tag{5}$$

In the same manner for a change in S, equation (4) will be

$$\frac{I''dS}{2S} \cdot \frac{\sqrt{S}}{\sqrt{S+R'}} \tag{6}$$

When R' is rather large in resistance, equation (4) becomes

$$\frac{I''dR_1}{2R_1} \cdot \frac{\sqrt{R_1}}{\sqrt{\frac{R'r_2}{R'+r_2} + R_1}} \tag{7}$$

In equation (7) the determining factor is $R'r_2/(R'+r_2)$. Equation (6) will be

$$\frac{I''dS}{2S} \cdot \frac{\sqrt{S}}{\sqrt{\frac{R'r_2}{R'+r_2} + S}} \tag{8}$$

One galvanometer, then, can be used for equations (7) and (8)

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16 F. E. Smith, "On the Method of High Precision of Comparison of Resistance", London Electrician, Vol. 57, p. 976.

VI Comparison of the Two Methods

In comparing the potentiometer and the Kelvin double bridge methods for the measurement of resistance, both methods must be operated under as nearly the same conditions as possible. The writer wishes to illustrate by a few concrete cases.

The double bridge used was the same as shown in figure 13, in which the b and b' arms consist of 25, 50, 100 ohms; a and a' are composed of five dials 10 x 100, 9 x 10, 9 x 1, 9 x 0.1, and 9 x 0.01ohms. This bridge was made by Otto Wolff, Berlin, Germany. The potentiometer was the slide wire type shown in Fig. 22. There are 14 x 0.5 ohm coils plus a slide wire having a resistance of 0.5 ohm. They are all adjusted to within 0.01% of one another. The total resistance of the potentiometer plus the rheostat is twenty ohms approximately.

(1) When ratio of R to S is nearly unity.

In the double bridge, a, a', b and b' are nearly all 150 ohms each.

(2) When R to S is nearly 1 to 10, a and a' are nearly 1000 ohms, while b and b' are about 100 ohms each.

In the potentiometer R is nearly the same as S, R' in Fig. 20 is 5 ohms. When R to S is 1 to 10, R' is 7.5 ohms.for S, and R is 0.75 ohms for R.

In the ideal case, when R = S nearly, the sensitiveness of the double bridge is proportional to

$$\frac{idR}{R}\ 0.005 \qquad \text{by equation (14)}$$

When a = a' = b = b' = 15 ohms, R = 0.01, S = 0.01.

When R to S = 1 to 10,

R = 0.001 ohm, S = 0.01 ohm.

a = a' = 1000 ohms. b = b' = 100 ohms.

The sensitiveness is proportional to

$$\frac{idR}{R}\ 0.000093$$

For the potentiometer method.

When R to S is unity

$$R = 0.01 \text{ ohm}, \quad S = 0.01 \text{ ohm}, \quad R' = 5 \text{ ohms}.$$

By applying formula (5) the method has a sensitiveness proportional to

$$\frac{I''dR}{R}\ 0.0169$$

When S is 10 times R

$$R = 0.001 \text{ ohm} \quad S = 0.01 \text{ ohm} \quad R' = 7.5 \text{ ohms for S}$$

$$R' = 0.75 \text{ ohm for R}$$

Sensitiveness proportional to

$$\frac{I''dS}{S}\ 0.055 \text{ for S}$$

and

$$\frac{I''dR}{R}\ 0.0066 \text{ for R.}$$

In the concrete case under which the writer was obliged to experiment, only one galvanometer was available for each method. The galvanometer used for the double bridge has

Period = 3.8 seconds

Resistance = 68.5 ohms

Megohm sensibility = 113 megohm - volts per 1 mm. deflection

Figure of merit $F = \frac{S_m}{T^2\sqrt{G}} = 194 \times 10^{-2}$.

The galvanometer used in the potentiometer work has

Period = 5.4 seconds

Resistance = 39.3 ohms

Megohm sensibility = 20.5 megohms - volt per 1 mm. deflection

Figure of merit $= \frac{S_m}{T^2\sqrt{G}} = 11.3 \times 10^{-2}$.

In the double bridge method

When

$$R/S = 1$$

$$g = 68.5 \text{ ohms}, \quad a = b = 150 \text{ ohms}, \quad R = 0.01 \text{ ohm}$$

and

$$S = 0.01 \text{ ohm} \quad i = 100 \text{ amperes},$$

the sensitiveness is proportional to

$$\frac{dR}{R} \; 0.23 \quad \text{from equation (21)}$$

When

$$R/S = 1/10$$

$$g = 68.5 \text{ ohms}, \quad a = 1000 \text{ ohms}, \quad R = 0.001 \text{ ohm}$$

$$S = 0.01 \text{ ohm} \quad i = 100 \text{ amperes},$$

the sensibility is proportional to

$$\frac{dR}{R} \; 0.00364 \quad \text{by equation (21)}$$

In the potentiometer method

When

$$R/S = 1$$

$$G = 39.3 \text{ ohms}, \quad R' = 5 \text{ ohms}, \quad r_2 = 20 \text{ ohms}, \quad R = S = 0.01 \text{ ohm}$$

$$I'' = 100 \text{ amperes}.$$

By modifying equation (4) we get

$$\frac{I''dR}{R} \cdot \frac{\sqrt{G}}{G + \left(\frac{R' \; r_2}{R' + r_2} + R\right)} \qquad (9)$$

The sensitiveness of the potentiometer is proportional to

$$\frac{dR}{R} \; 1.4$$

When

$$R/S = 1/10$$

$$G = 39.3 \text{ ohms}, \quad R' = 7.5 \text{ ohms}, \quad R = 0.001 \text{ ohm}$$

$$S = 0.01 \text{ ohm}, \quad r_2 = 20 \text{ ohms}, \quad I'' = 100 \text{ amperes},$$

the sensitiveness is proportional to

$$\frac{dR}{R} \; 1.28$$

As is shown by these figures the potentiometer method is much superior in sensibility than the double bridge method under the similar circumstances. In the measurement of resistances ranging from 1 ohm to 0.001 ohm, a probable error of ±0.005% is allowed; of resistances less than 0.001 ohm, an accuracy greater than 0.02% can not be expected with good speed. To measure such small resistances either the double bridge method or the potentiometer method may be employed, using heavy current through the resistances. The choice of one method rather than the other depends upon the nature of the current, construction of the resistances, and the ability of the observer. The writer lists the advantages and the disadvantages of the two methods.

Advantages of the Kelvin Double Bridge Method

1 Rapidity in manipulation.

2 Unsteady current can be used.

3 Thermo e.m.f. can easily be taken care of.

4 Ratio of 1 to 10 can be compared the same as unity ratio.

5 Only one reading is necessary.

Disadvantages of the Kelvin Double Bridge Method

1 The resistance of the potential leads must be known, in some cases it is impossible to determine their values. See Fig. 27.

2 The connecting link C between known and unknown must be as small a resistance as possible. Often this requirement was found impossible.

3 The sensitiveness is less than the potentiometer method. See Tables VII, X, and XI.

4 When a is not exactly equal to a' and b differs from b', they must be adjusted once for all, otherwise an error of 0.01 and 0.04% is introduced. See Tables VIII and IX for illustration.

Advantages of the Potentiometer Method

1 The Potentiometer method is a sensitive method.

2 The resistance of the drop leads does not affect the measurements.

3 Any construction or shape of shunt can be measured. See Table VII.

4 Whatever the resistance of standard used, the resistance of unknown is read directly.

Disadvantages of the Potentiometer Method

1 The source of current must be constant; otherwise, the error will be too large.

2 The thermo e.m.f. of the circuit can not be easily taken care of.

3 More than two readings are necessary.

4 It is slow compared to the double bridge method.

5 When the ratio of known and unknown resistances is greater than 2 to 1, accuracy is reduced by inverse ratio. See Tables VIII and IX for the illustration.

CONCLUSION

On the Kelvin double bridge, five figures are read directly, and the next place can be calculated from the galvanometer deflection. On the potentiometer five figures are read directly, and the next figure can be estimated from the last scale division. The two instruments are able to indicate a difference of .00001 ohm between the two resistances measured. Allowing all possible errors, both methods can be used with an accuracy of 0.005% in unity ratio, and 0.01% in 1/10 ratio. Heavy current may be used in either method. A skilled hand or novice can manipulate a Kelvin bridge with ease if the construction of the resistances is suitable for the method, and all the corrections are known. To manipulate a potentiometer requires experience and much skill, although any form of resistance can be measured with it. When accuracy is demanded for certain measurements more than time element, then the potentiometer method is suitable in all cases. When rapidity is demanded besides accuracy, the Kelvin double bridge method should be used in limited instances. Table VI illustrates errors that appeared in reversing the arms of the Kelvin bridge. Tables VIII and IX show the series of measurements on several shunts by the double bridge and by the potentiometer methods.

Most of the above work was done at Schenectady, New York, where the writer has been in the employ of the General Electric Company, but the direction of the work has been by Professor A. P. Carman of the University of Illinois, to whom the writer extends his thanks for his kindly interest and suggestions offered from time to time.

Laboratory of Physics
University of Illinois
June, 1914

TABLE I

CHECKING WOLF-"KELVIN" BRIDGE

#A

Upper Dial		Lower Dial		Calculated Relation Between Upper & Lower Upper Coils as Std.		Observed Relation Between Upper & Lower Upper Coils as Std.	
Coil	Observed Resistance	Coil	Observed Resistance				
			Tenths (.1) Ohm Coils				
	Ohms		Ohms	Low. Dial	%		%
1	.099869	1	.099884	.015	high	.013	high
2	.09998	2	.099976	.004	low	.005	low
3	.099977	3	.099993	.016	high	.012	high
4	.099972	4	.099978	.006	high	.005	high
5	.099977	5	.099972	.005	low	.01	low
6	.099986	6	.099961	.025	low	.025	low
7	.09999	7	.099978	.012	low	.01	low
8	.09997	8	.099995	.025	high	.024	high
9	.0100078	9	.100067	.011	low	.005	low
Tot.	.899798	Tot.	.899805	.007	high		
Ob.Tot.	.89993	Tot.Obs.	.89996	.003	high		
			Unit (1.) Ohm Coils				
	Ohms		Ohms	Low Dial	%		%
1	1.00011	1	1.00003	.008	low	.008	low
2	.99999	2	1.00001	.002	high	.001	high
3	1.00003	3	1.00023	.02	high	.015	high
4	1.00005	4	.99988	.017	low	.013	low
5	1.00012	5	1.0001	.002	low		
6	1.00006	6	1.00002	.004	low	.002	low
7	1.00020	7	1.00011	.009	low	.005	low
8	1.00018	8	.99997	.021	low	.015	low
9	1.00021	9	1.00002	.019	low	.009	low
Tot.	9.00095	Tot.	9.00037	.006	"		
Ob.Tot.	9.001	Obs.Tot.	9.0009	.001	"		

TABLE I-2

Upper Dial Coil	Observed Resistance	Lower Dial Coil	Observed Resistance	Calculated Relation Between Upper & Lower Upper Coil As Std.		Observed Relation Between Upper & Lower Upper Coil As Std.	
			Ten ohm Coils.				
	Ohms		Ohms	Low Dial	%		%
1	10.0002	1	10.0003	.001	high	.005	high
2	9.9995	2	9.9997	.002	"	.001	"
3	10.0002	3	1.0002	----	----	.001	"
4	9.0004	4	10.0000	.006	high	.005	"
5	10.0005	5	10.0007	.002	"	---	----
6	10.0005	6	10.0003	.002	low	.001	low
7	10.0000	7	10.0001	.004	"	.006	"
8	10.0000	8	9.9997	.003	"	.003	"
9	10.0014	9	10.0003	.001	"	.009	"
Tot.	90.0022		90.0013	.001	"		
Obs.Tot.	90.023		90.017	.007	"		
			Hundred Ohm Coils				
	Ohms		Ohms	Low Dial	%		%
1	99.978	1	99.982	.004	High		
2	99.983	2	99.984	.001	"	.002	low
3	99.982	3	99.986	.004	"	.001	high
4	99.981	4	99.984	.003	"	.001	"
5	99.983	5	99.984	.001	"		
6	99.982	6	99.984	.002	"	.001	high
7	99.991	7	99.983	.008	low	.004	low
8	99.991	8	99.985	.006	"	.005	"
9	99.984	9	99.982	.002	"	.001	high
10	99.985	10	99.983	.002	"		
Tot.	999.84		999.837	.0003	"		

TABLE I - 3

COMPARING RESISTANCE OR
VALUE OF 2-PLUG COIL SETS

Plug b Taken as Standard	b' Plug Resistance Coils	Difference Taking b as Standard
Ohms	Ohms	
25	25.004	.016% high
50	50.005	.01% "
100	100.002	.002% "
b	a'	
25	25	0
50	50	0
100	100.001	.001%
b	a	
25	25.003	.012
50	50.006	.012
100	100.001	.001

TABLE II.

TABLE SHOWING DIFFERENCE IN VALUE OF RESISTANCE CAUSED BY USING EQUAL OR UNEQUAL POTENTIAL DROP LEADS.

Standard of Comparison 1001	Resistance Measured 1005	Condition of Leads: Outside	Inside	Inside	Outside	Value of Resistance	Difference
		Value of b and b' arms = 150 Ohms					
0.1 Ohm		.006	.006	.006	.006	.099995	
							$\frac{4}{100,000}$
"	"	.032	.032	.032	.032	.099991	
							$\frac{21}{100,000}$
"	"	.032	.032	.006	.006	.100012	
							$\frac{26}{100,000}$
"	"	.006	.006	.032	.032	.099984	
"	"	.032	.006	.006	.006	.100020	
							$\frac{66}{100,000}$
"	"	.006	.006	.006	.032	.099954	
"	"	.006	.032	.006	.006	.099995	
							$\frac{2}{100,000}$
"	"	.006	.006	.032	.006	.099997	

Accurate to within .005%

All the measurements were made at 22°C. Value of C = .001 Ohm.

and are connected to a and a' respectively, while and are connected to b and b' respectively, while and are called outside leads, while and are called inside leads.

When R is nearly equal to S, b and b' are nearly equal to a and a' respectively, or in the neighborhood of 150 Ohms. Then .006 Ohm is $\frac{4}{100,000}$ of 150 ohms and .032 is $\frac{21}{100,000}$ of 150 ohms.

$(.032 - .006)/150 = \frac{17}{100,000}$ of 150 ohms.

TABLE III

TABLE SHOWING DIFFERENCE IN VALUE OF RESISTANCE MEASURED WHEN DIFFERENT VALUE OF CONNECTION FOR ( )

Accurate to within .005%

Standard of Comparison	Resistance Measured	Value of Standard	Value of Standard	Value of Resistance	Difference	
		Value of b and b' = 100 ohms				
#1001 "N"	#1005 "X" .1 ohm	.001ohm	.099994		1/100,000	3/100,000
		Value of b and b'axm = 150 ohms				
#1001 "N"	#1005"X" .1 ohm	.001	.099993		4/100,000	
	Position of #06354 and #05615 reversed					
#1001 "X"	#1005"N" .1 ohm	.001	.099997		7/100,000	1/100,000
	Original Position b and b' = 150 ohms					
#1001 "N"	#1005"X" .1 ohm	.002	.099990		8/100,000	
	Position Reversed					
#1001 "X"	#1005"N" .1 ohm	.002	.099998		3/100,000	0
	In Original Position					
#1001 "N"	#1005 "X" .1 ohm	.0005	.099995		3/100,000	
	Position Reversed					
#1001 "X"	#1005 "N" .1 ohm	.0005	.099998		8/100,000	0
	In Original Position					
#1001 "N"	#1005 "X" .1 ohm	.032	.099990		8/100,000	
	Position Reversed					
#1001 "X"	#1005 "N" .1 ohm	.032	.099998		11/100,000	18/100,000
	In Original Position					
#1001 "N"	#1005 "X" .1 ohm	.15	.099987		7/100,000	
	Position Reversed					
#1001 "X"	#1005 "N" .1 ohm	.15	.099980			

Drop leads and = .006 ohms.

Measurement made at room temperature 24°C

When R and S are nearly equal and b and b' = 100, and a and a' nearly equal to 100. When b and b' were made 150 each a and a' will be nearly 150 ohms.

TABLE IV.

POTENTIOMETER METHODS.

MEASUREMENTS SHOWING EFFECT OF VARIATION OF CURRENT IN TWO CIRCUITS.

Readings on Standard	Average	Readings on Shunt		Resistance Standard & Value	Shunt	Restistance Shunt
				01040 = .0009983	#06320	
10.-		1.0304				
9.99575		1.0306				00.010282
9.9956	9.9955	1.0295	1.0298			
9.9925		1.0294				
9.9918						
10.-		1.0302		0104 = .0009983		
9.9966		1.0300			#06370	
9.9942	9.9940	1.0298	1.0297			.00010283
9.9904		1.0290				
9.9887						

TABLE V.

POTENTIOMETER METHOD.

Setting	Reading	Average Reading	Corrected Reading	Standard Used	Resistance of Standard	Resistance of Shunt
10.	10.0085					
10.	10.0084	10.0085	10.0085	#05615	.099985	.10007
10.	10.0085					
10.	10.0084					
10.	10.0142					
10.	10.0141	10.01415	10.0145	#05480	.0100007	.0100149
10.	10.0142					
10.	10.0142					
10.	2.4986					
10.	2.4985			#05480	.0100007	.0024988
10.	2.4987	2.49856	2.49866			
10.	2.4985					
10.	2.4985					
10.-	10.0068					
10.-	10.0070	10.0069	10.0069	#9 5778	.00099988	.00100054
10.-	10.0070					
10.-	10.0069					

TABLE VI.

2 - 1 ohm = 1 ampere RESISTANCE STANDARDS.

COMPARED EACH OTHER BY THREE METHODS.

CAREY-FOSTER, KALVIN DOUBLE BRIDGE,
AND POTENTIOMETER METHODS.

Standard used	Resistance compared	By Carey-Foster. ohm	By Kalvin Double Bridge ohm	By Potentiometer Method ohm	Value of Standard assumed. ohm
20025					1.00011
	#20026	1.00009	1.00021	1.00009	
		1.00008	.99998	1.00009	
Difference in %		.001%	.023 %	.000 %	

By using same amount of current.

On Carey-Foster Bridge, difference of 1/100,000 can be read directly.

On Potentiometer, difference of 1/100,000 can be estimated.

On Kelvin Bridge, difference of 5/100,000 can be read directly.

and means the positions of 20025 and #20026 are reversed.

These three Sets of Measurements employing three methods were made to check accuracy of Kelvins Bridge.

TABLE VII.

CURRENT CARRYING RESISTANCE STANDARD
#03020 WAS COMPARED AGAINST #20025.
BY KELVIN DOUBLE BRIDGE AND POTENTIOMETER METHODS.

I. By Potentiometer Method.

Formula $X = \frac{\text{Reading}}{\text{Setting}} S_d$

Standard	Value of Stand.	Setting	X	Reading	Correction	Corrected Reading	Value of X	Temp.	Current
200025	1.00011	10.0011	03820	10.0002 10.0002 10.0002 10.0002	0	mean 10.00002	1.00002	21°C	.9 amp.

Difference among four Readings none.

II. By Kelvin Double Bridge.

$X = N \frac{a^1}{b^1}$ Where a^1 is a corrected; b^1 is b corrected

Temp.	b	b^1	a	a^1	N	Value of N	X	Value of N	Current
21°C	150	150.013	149.943	149.956	20025	1.00011	03820	.99973	.gamp
21°C	150	150.013	149.946	149.959	20025	1.00011	03820	.99975	"
							Mean	.99974	

Difference between two readings = $\frac{2}{100,000}$ = .002 %

" " Kelvin and Potentiometer Measurements =

$\frac{26}{100,000}$ or .026 %

TABLE VIII.

CURRENT CARRYING.
RESISTANCE STANDARD MEASURED BY KELVIN
DOUBLE BRIDGE AND POTENTIOMETER METHODS.
AT ROOM TEMPERATURE 22°C.

Resistance No. of.	Date.	Measured by Potentiometer Method.	Measured by Double Bridge Method.	Difference between Methods. %
	1913			
443	2 - 17	.009998 ohm.	.0099986 ohm.	.006
"	4 - 30	.009999	.0099996	
Difference		.01%	.02%	
x 444	2 - 17	.0009983	.00099828	.02
"	4 - 30	.00099834	.00099876	
Difference		.004%	.048%	
182093	2 - 17	.099990	.099987	.002
"	4 - 30	.099987	.099995	
Difference		.003%	.007%	
91454	2 - 17	.100010	.100008	.005
"	4 - 29	.100010	.100021	
Difference		0	.015%	
181653	2 - 17	.0100007	.0100010	.009
"	4 - 29	.0100000	.0100016	
Difference		.007%	.006%	
182062	3 - 10	.00099988	.00099985	.011
"	10 - 3	.0009998	.00099978	
Difference		.008%	.007%	
279646	1 - 14	.0100033	.0100035	.012
"	10- 31	.0100022	.0100045	
Difference		.011%	.01%	
279645	1 - 10	.099982	.099979	.002
"	4 - 29	.09998	.099986	
Difference		.002%	.007%	
18717	10- 30	.99983	- - -	
"	1914			
	2 -27	.99990	.99978	.009
Difference		.007%		
	1913			
91316	10 -30	.99992		
"	1914			
	2 -27	1.00002	.99984	.013
Difference		.01%		

Table IX.

MEASUREMENTS MADE ON CURRENT CARRYING STANDARD SHUNTS.

#	Current Capacity	Date 1913 & 1914	Measured by Kelvin double Bridge Method.	Measured by Potentiometer Method.	Difference between two readings (average) of each Method.		
	Amperes		ohm.	Ohm.	Kelvin. %	Pot. %	Kelvin. to Pot.
		1914					
182047	1000	1 - 3	.00050461		.054		
"		1 - 9	.00050433				
182110	2000		.00025112	.00625114			
"			.00025117	.00025115	.02	.004	0
182127	1500		.00036004	.00030983			
"			.00036002	.00035979	.006	.012	.046
182006	100		.0049925	.00499257			
"			.0049920	.0049938	.01	.02	.016
182005	50		.0100143	.0100145			
"			.0100143	.0100144	0	.001	.001
182003	10		.050060	.050061			
"			.050060	.050063	0	.004	.004
182002	5		.100155	.100160	.005	.003	.001
"			.100160	.100157	.		
182132	1 1/2		.33341	.33337	.003	.003	.012
"			.33340	.33336			
182001	5		.100078	.100076	.002	.001	.002
			.100080	.100077			
182004	20		.025068	.025067	.008	.004	.004
			.025070	.025066			
182128	200		.0249969	.0024992	.006	.016	.014
			.0249983	.0024996	.		

TABLE X

TABLE SHOWING VARIATION IN MEASUREMENTS

When errors in a^1 & b^1 arms or lead resistances or contact errors were not taken care of

$$X = N \frac{a^1}{b^1}$$

N	Resistance of N	X	b^1	a^1	Resistance of X	Variation from mean value
182062	.00099957	182045	100	250.1	.0024999	- .032
"	"	"	175	437.5	.0024989	- .072
"	"	"	125	312.8	.0025013	+ .034
"	"	"	150	375.4	.0025022	+ .060
		mean value		.0025007		

By Potentiometer Method

Stand.	Resistance of Standard	Setting	X	Reading	Correction.	Cor. Reading.	Resistance of X
181653	.0099957	14.9995	182045	3.7522			
				3.7523			
				3.7524	.0003	3.7520	
				3.7523			.00250153
			Mean	3.7523			

Variation ' Potentiometer reading is ± .003%

TABLE XI

TABLE SHOWING VARIATION IN MEASUREMENT BY KELVIN DOUBLE BRIDGE WHEN ERROR IN a^1 or b^1 ARM OR RESISTANCE OF LEADS WERE NOT TAKEN CARE OF

$$X = N \frac{a^1}{b^1}$$

N	Resistance of Stand	a^1	b^1	X	Resistance of X First	Resistance of X Second	Difference
181653	.0099997	100	1000.6	182001	.100057	.100057	0
			1000.6				
"	"	"	1001.3	182002	.100127	.100137	.01%
			1001.4				
"	"	"	5004.	182003	.050038	.050048	.02%
			5005.				
"	"	"	250.6	182004	.025059		
		175	438.6			.025062	.012%
182062	.00099957	"	1001.9	182005	.0100147		
			1001.6			.0100116	.03%
"	"	"	499.4	182006	.00499285		
"	"	175	873.9		.0	.0049915	.027%
"	"	50	249.8	182006	.0049944		.03%
		175	434.9			.02484	
"	"	100	249.5	182045	.0024939		.04%

$$N = \frac{b^1}{a^1} X$$

X				N			
181653	.0099997	100	999.1	182046	.00100087		.043%
			998.6			.00100130	
"	"	50	997.6	182047	.00050119		
			996.5			.00050174	.11%
"	"	25	700.6	H	.00035683		
			70.04		0	.00035692	.027%

△ Shunt, #182045 was highly heated by over load.

TABLE XII

CHECKING THE COILS AND SLIDE WIRE OF A POTENTIOMETER

No. of Coil	Deflection in milli-meters	No. of Coil	Deflection in millimeters
1	3 High	1	2.5 High
2	1.5 "	2	1. "
3	2. Low	3	1.5 Low
4	0	4	1.5 High
5	2.5 Low	5	1. Low
6	2.5 "	6	2. "
	Std.		Std.
7	0	7	0
8	0	8	0
9	1.5 Low	9	1. Low
10	3. "	10	2. "
11	2. "	11	1.5 "
12	1 "	12	1. "
13	1.5 "	13	2. "
14	1.5 "	14	1.5 "
S. W.	1. High	S. W.	2.5 High
	1 mm = 0.002%		1 mm = 0.003%

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